ECONOMETRIC APPROACH TO THE DEMAND FUNCTION

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Abstract

Econometric methods are applied in the analysis of many phenomena in the economic reality, such as production, costs, and supply search.

Valuable results are found in certain limits and under certain conditions by using functions.

Synergy of statistical data and the application of mathematical methods will be used to analyze the demand function.

Key words: *demand function, econometric methods, market, supply function*

JEL Classification: C4, C5, C54

1. INTRODUCTION

Research into the forms of the demand function enables us to deal with and solve complex economic problems. This paper investigates the demand function without highlighting aspects of demand or differentiating demand for certain products on the market or certain groups of consumers.

The demand for a product depends on many variables, such as the price of the observed product, its substitutes, as well as other products on the market.

Furthermore, the demand depends on the purchasing power, consumer tastes, the time factor, but also on many other factors that directly and indirectly affect the demand for products.

The demand for a product can be expressed by the following mathematical expression (Chiang A.C.;1994, p.64): $q = T(p, p_1, ..., p_n, x_1, x_2, ..., x_m, k, t)$, where q is the quantity of the desired product, p is the price of the desired product, $p_1, ..., p_n$ are prices of other products on the market, $x_1, ..., x_m$ are other factors affecting the market, k is consumer income, and t is time.

Let the function of the quantity of the product q be a first derivative with respect to the variable p (the price of the desired product) in the time interval $\langle c, d \rangle$ in which the demand is observed; then the condition $\frac{\partial T}{\partial p} < 0$ must be satisfied in the time interval $\langle c, d \rangle$.

Knowing that the first derivative refers to the change in the speed, we may conclude that the demand decreases if the price rises in free market conditions.

In applications, the results are satisfactory if, under certain conditions, the demand is considered only as a function of price, i.e., $q = T(p)$.

If $T(p)$ is a derivative of the interval $\langle c, d \rangle$, then $\frac{\partial T}{\partial p} < 0$, knowing that the partial derivative and the differential for functions of one variable are equivalent.

Furthermore, the demand function q must also satisfy the following conditions:

Interval $\langle c, d \rangle$ must be finite and approximately equal to the interval of empirical values, q must be a positive function of the price p. Hence, it must be $p > 0$ and $q > 0$ in the interval $\langle c, d \rangle$.

Indeed, the analytical form of the function $q = T(p)$ often varies and depends on empirical values of p and q.

The forms of demand functions are usually simple because of the amount of empirical data and due to the complexity of determining the parameters of the function itself.
The most common forms of demand models are as follows:

a) $q=-ap+b$,
b) $q=-ap^2+b$,
c) $q=\frac{1}{ap+b}$,
d) $q=\sqrt{-ap+b}$,
e) $q=-a\sqrt{p}+b$,
f) $q=\frac{a}{p^n}+b$, and
g) $q=ae^{-bp}$,

where a and b are positive constants for a certain time interval.
Parameters a and b are found from the empirical demand values, provided that the sum of the squared deviations of empirical demand values from the corresponding values on the selected curve is minimal, i.e., by the least squares method (Scitovski R. et al;1995, p.102).
Mathematical analogy yields an economic supply function.
Since mathematics is an approximate science, we can ignore all variables except the price of a product, and the supply function becomes $P=q(p)$.

In the interval $\langle c,d\rangle$, which corresponds to the empirical data, $p>0$ and $q>0$.

The supply function is an increasing function, so $\frac{\partial P}{\partial p}>0$.

Supply and demand for a product on the free market determine its price.
Market price is determined by equalizing demand and supply functions cijenu (Kmenta J.;1997, p.722).
Hence, $T(p)=P(p)$ is the condition of equilibrium.
A realistic solution p^* in the interval $\langle c,d\rangle$ in which the functions are analyzed is unique, since one function is decreasing and the other one is increasing in the interval $\langle c,d\rangle$.
Note that outside the interval $\langle c,d\rangle$ there may be more sections that have no economic significance and thus they are not taken into consideration.

According to the aforementioned, we may conclude that a product market model is given by the demand function, the supply function and the balance equation.

2. DEMAND WITH MULTIPLE VARIABLES

Consider multiple products on the market, e.g., n products $Q_1, Q_2, ..., Q_s, ..., Q_n$, and let us pay attention to the product Q_s.

Demand for the product Q_s is actually a function of the prices of all products, if we consider the dependence of demand on prices and not on other factors.

Denote by q_s and $p_1, p_2, ..., p_n$ the amount of demand for the product Q_s and the prices of products $Q_1, Q_2, ..., Q_n$, respectively.

The demand function for the product Q_s can be taken as an unambiguous function of prices p_s.

The system of interdependence between prices and the corresponding amount of demand for $s = 1,2,...,n$, can be written as (Gruić B. et al; 2006, p.76)

$$\begin{aligned} q_1 &= T_1(p_1, p_2, ..., p_n) \\ q_2 &= T_2(p_1, p_2, ..., p_n) \\ &\vdots \\ q_n &= T_n(p_1, p_2, ..., p_n) \end{aligned}$$

Similarly to the previous system, we can observe the system of demand functions in inverse form if there is a solution with respect to p_s, $s = 1,2,...,n$, i.e.,

$$\begin{aligned} p_1 &= T_1^{-1}(q_1, q_2, ..., q_n) \\ p_2 &= T_2^{-1}(q_1, q_2, ..., q_n) \\ &\vdots \\ p_n &= T_n^{-1}(q_1, q_2, ..., q_n) \end{aligned}$$

Example 1.

Consider a system of affine demand functions

$$\begin{aligned} q_1 &= a_{11}p_1 + a_{12}p_2 + ... + a_{1n}p_n + b_1 \\ q_2 &= a_{21}p_1 + a_{22}p_2 + ... + a_{2n}p_n + b_2 \\ &\vdots \\ q_n &= a_{n1}p_1 + a_{n2}p_2 + ... + a_{nn}p_n + b_n \end{aligned},$$

where $a_{ss} \le 0,\ s = 1,2,...,n$, that can be written in matrix form

$$\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix},$$

i.e., $q = Ap + b$, where q, p and b are $n-$dimensional vectors of demand, prices and free members, respectively, and the square matrix A is the coefficient matrix of the system of $n-$th order.

From the previous matrix equation, for a given demand vector, we find $p = A^{-1}(x-b)$, where A^{-1} is the inverse demand coefficient matrix and $|A| \neq 0$.

The expression $q = Ap + b$ gives a relationship between market prices and the corresponding demand for the products under consideration.

3. DETERMINATION OF DEMAND FUNCTION PARAMETERS

Let the demand function be given in form of a polynomial:
$q = a_1 p^{k-1} + a_2 p^{k-2} + \ldots + a_{k-1} p + a_k$, where k parameters should be defined.
Suppose an empirical demand function is given in an affine form $q = ap + b$, where parameters a and b are unknown.

According to the least squares method (Scitovski R. et al; 1994, p.182) (looking vertically), the sum of the squared deviations of empirical demand values from the corresponding values on the line tends to minimality, i.e., $\sum_{i=1}^{n}(q_i - \hat{q}_i)^2 \to \min$.

Since by assumption points (p_i, q_i), $i = 1,2,\ldots,n$ lie on the line, they satisfy the equation $q = ap + b$.

After the substitution in $\sum_{i=1}^{n}(q_i - \hat{q}_i)^2 \to \min$, we have $T(a,b) = \sum (q_i - ap_i - b)^2$.

Partial derivatives of the previous function are as follows:

$$\frac{\partial T}{\partial a} = 2\sum_{i=1}^{n}(q_i - ap_i - b)(-p_i), \quad \frac{\partial T}{\partial b} = 2\sum_{i=1}^{n}(q_i - ap_i - b)(-1).$$

By putting $\frac{\partial T}{\partial a} = \frac{\partial T}{\partial b} = 0$, and after summing, we get the following normal equations:

$$\begin{aligned} \sum q_i &= a\sum p_i + nb \\ \sum q_i p_i &= a\sum p_i^2 + b\sum p_i \end{aligned},$$

such that solving with respect to a and b yields

$$\begin{aligned} a &= \frac{n\sum p_i q_i - \sum p_i \sum q_i}{n\sum p_i^2 - \left(\sum p_i\right)^2} \\ b &= \frac{\sum q_i - a\sum p_i}{n} \end{aligned},$$

where n is the number of data.
Function $T(a,b)$ has a minimum value if the following condition is satisfied (Barnett et al.; 2006, p.379)

$$\frac{\partial^2 T}{\partial a^2} > 0,\ \frac{\partial^2 T}{\partial b^2} > 0,\ \left(\frac{\partial^2 T}{\partial a \partial b}\right)^2 - \frac{\partial^2 T}{\partial a^2}\frac{\partial^2 T}{\partial b^2} < 0\,.$$

We have $$\frac{\partial^2 T}{\partial a^2} = 2\sum p_i^2 > 0,\ \frac{\partial^2 T}{\partial b^2} = 2n > 0$$

$$\frac{\partial^2 T}{\partial a \partial b} = 2\sum p_i \text{, and}$$

$$\frac{\partial^2 T}{\partial a \partial b} - \frac{\partial^2 T}{\partial a^2}\frac{\partial^2 T}{\partial b^2} = 4\left(\sum p_i\right)^2 - 4n\sum p_i^2 = -4\left[n\sum p_i^2 - \left(\sum p_i\right)^2\right] = \frac{-4}{n^2}\left[\frac{\sum p_i^2}{n} - \overline{p}^2\right] =$$

$\frac{-4}{n^2}\sigma_p^2 - \left(\frac{2}{n}\sigma_p\right)^2 < 0,$ where σ_p^2 is price variance.

Let us note the symmetry in forming the normal equations that can be used for other forms of demand.

By summing equations $q_i = ap_i + b$ with respect to i, we have

$$\sum q_i = a\sum p_i + nb\,.$$

Then by multiplying by p_i and summing with respect to i, we obtain

$$\sum q_i p_i = a\sum p_i^2 + b\sum p_i\,,$$

which leads to finding parameters a and b.

The previous procedure can be carried out for the formation of normal demand equations

$$\hat{q} = a_1 p^{k-1} + a_2 p^{k-2} + \ldots + a_{k-1} p + a_k\,,$$

bearing in mind

$$\frac{\partial T}{\partial a_s} = 2\sum\left(q_i - \hat{q}_i\right) p_i^{k-s} = 0,\ s = 1,2,\ldots,k$$

$$\sum p_i^{k-s} q_i = \sum \hat{q}_i p_i^{k-s}\text{, i.e.,}$$

$$\sum p_i^{k-s} q_i = \sum\left[a_1 p_i^{k-1} + a_2 p_i^{k-2} + \ldots + a_k\right] p^{k-s}\text{, i.e.,}$$

$$\sum p_i^{k-s} q_i = \sum\left[a_1 p_i^{2k-s-1} + a_2 p_i^{2k-s-2} + \ldots + a_k p_i^{k-s}\right],\ s = 1,2,\ldots,k\,.$$

CONCLUSION

It can be said that mathematical methods have been used in economics for a long time. One of the reasons lies in the difference that exists between dependence in economics and dependence in natural sciences where mathematics has been largely used.

Today, the application of mathematical methods in economic research is characterized by the emergence of mathematical models.

The very selection of models and their interpretation are studied by operations research, linear and nonlinear programming, econometric analysis, etc.

Today, significant solutions in optimization of many economic phenomena are found thanks to mathematics.

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