THE INFLUENCE OF CONFIDENCE LEVEL, CORRELATION AND VOLATILITY ON VALUE AT RISK. SIX CASE STUDIES

Izabela Pruchnicka-Grabias, Ph.D.[1]
[1]Warsaw School of Economics, Republic of Poland, ipruch@wp.pl

Abstract

Studies show that hedge funds and other financial institutions often apply the standard deviation as a risk measure. Even if one looks at hedge fund internet pages with investment results data for investors, they usually present them with standard deviations and Sharpe indicators, neglecting the fact that their investment assets are not always normally distributed, as well as such important measures as for example kurtosis and skewness. The author estimates the correlation and volatility for selected investment assets and verifies assumptions of popular risk models concerning these parameters. The impact of the confidence level, correlation and volatility on Value at Risk is analyzed.

JEL Classification: E44, O16

Keywords: hedge funds, financial institutions, standard deviation, investment assets

INTRODUCTION

Studies show that hedge funds and other financial institutions often apply the standard deviation as a risk measure. Even if one looks at hedge fund internet pages with investment results data for investors, they usually present them with standard deviations and Sharpe indicators, neglecting the fact that their investment assets are not always normally distributed, as well as such important measures as for example kurtosis and skewness defined by the following formulas.

$$\text{Kurtosis} = \frac{T(T+1)}{(T-1)(T-2)(T-3)}\sum_{t=1}^{T}\left(\frac{R_{t-1,t}-\overline{R}}{\sigma}\right)^4 - \frac{3(T-1)^2}{(T-2)(T-3)},$$

where:

T – the number of observations

σ – the standard deviation of rates of return

$\bar{R}$ – arithmetic mean of rates of return[1]

Skewness is the third central moment of a distribution and measures the symmetry of a return distribution around the mean. Mathematically it is calculated as:[2]

$$\text{Skewness} = \frac{T}{(T-1)(T-2)}\sum_{t=1}^{T}\left(\frac{R_{t-1,t}-\bar{R}}{\sigma}\right)^{3}$$

Taking the assumption that the volatility does not change in time leads to inadequate results. It has been well documented in the literature that it changes. Thus, it also influences value at risk. The key matter for calculating VaR is the choice of the holding period. Holding period is understood as the period in which the calculated loss may be generated. The proper choice of the holding period must be based on the time in which an institution is able to sell the majority of liquid assets. Banks usually use a one day holding period, however for hedge funds it is not adequate, because their assets are less liquid. For this reason, they often use a 10-day holding period.

Another weak point of this method is using the square root of time. Although it is widely accepted in practice, F. Duc and Y. Schorderet[3] show that the approximation of VaR using the square root of time rule differs significantly from the correct VaR and makes it impossible to catch the fact that the risk starts to fall down from the sixth month.

Value at risk in the given confidence interval can be written as:[4]

VaR = V × P × S
where:

[1] F.S. Lhabitant, Handbook of hedge funds, John Wiley & Sons, Ltd., Chichester 2006, p. 437. Note that some analysts do not subtract the second term from the kurtosis. As a result, when T is large, the threshold value for the normal distribution becomes 3 rather than 0.

[2] F.S. Lhabitant, Handbook of hedge funds, John Wiley & Sons, Ltd., Chichester 2006, p. 436 – 437.

[3] F. Duc, Y. Schorderet, Market Risk Management for Hedge Funds.Foundations of the Style and Implicit Value-at-Risk, John Wiley & Sons, Ltd., Chichester 2008, p. 163 – 164.

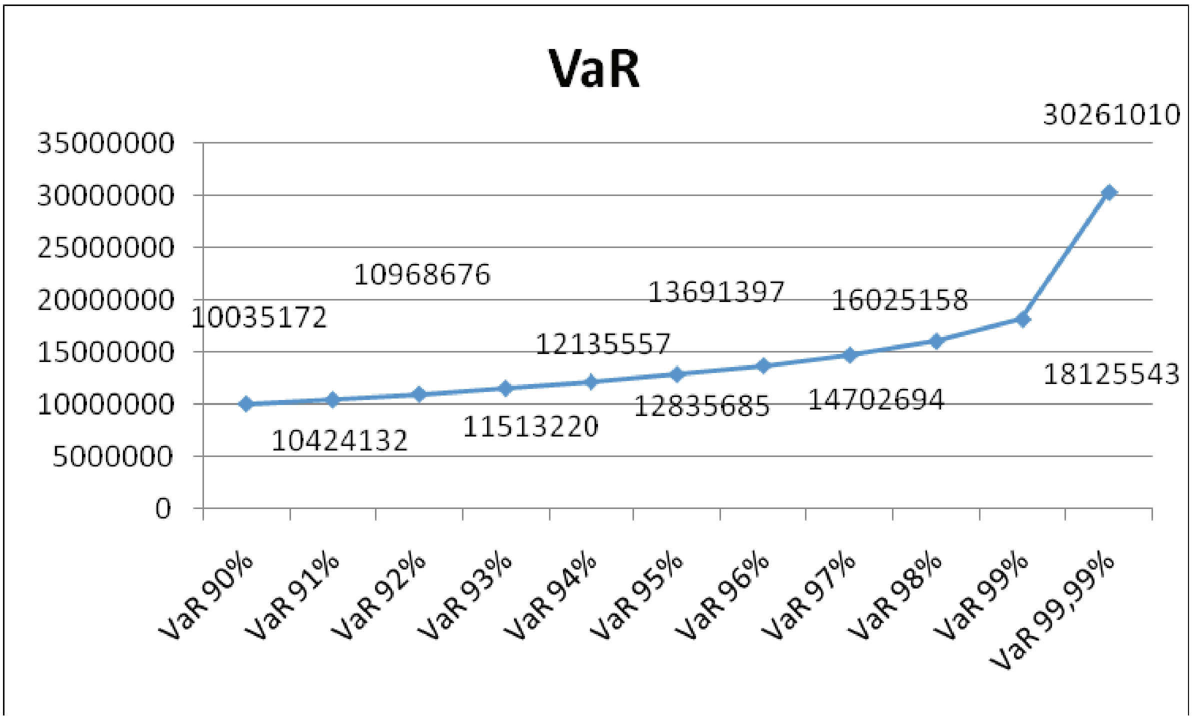
[4] P. Best, Wartość narażona na ryzyko. Obliczanie i wdrażanie modelu VaR [Value at risk. Calculating and implementing the VaR model], Dom Wydawniczy ABC, Kraków 2000, p. 27.

V – variance of rates of returns of the asset
P – the value of the investment
S – the number of standard deviations below the average

Case study I

Let's assume that a hedge fund invests its assets of 100000000 USD in crude oil futures. Let's calculate value at risk for different confidence levels for a 10 – day holding period, provided that the daily volatility is counted for the period 2005 – 2010 (see chart 1).

Chart 1. The impact of the confidence level on VaR changes for crude oil daily rates of return.

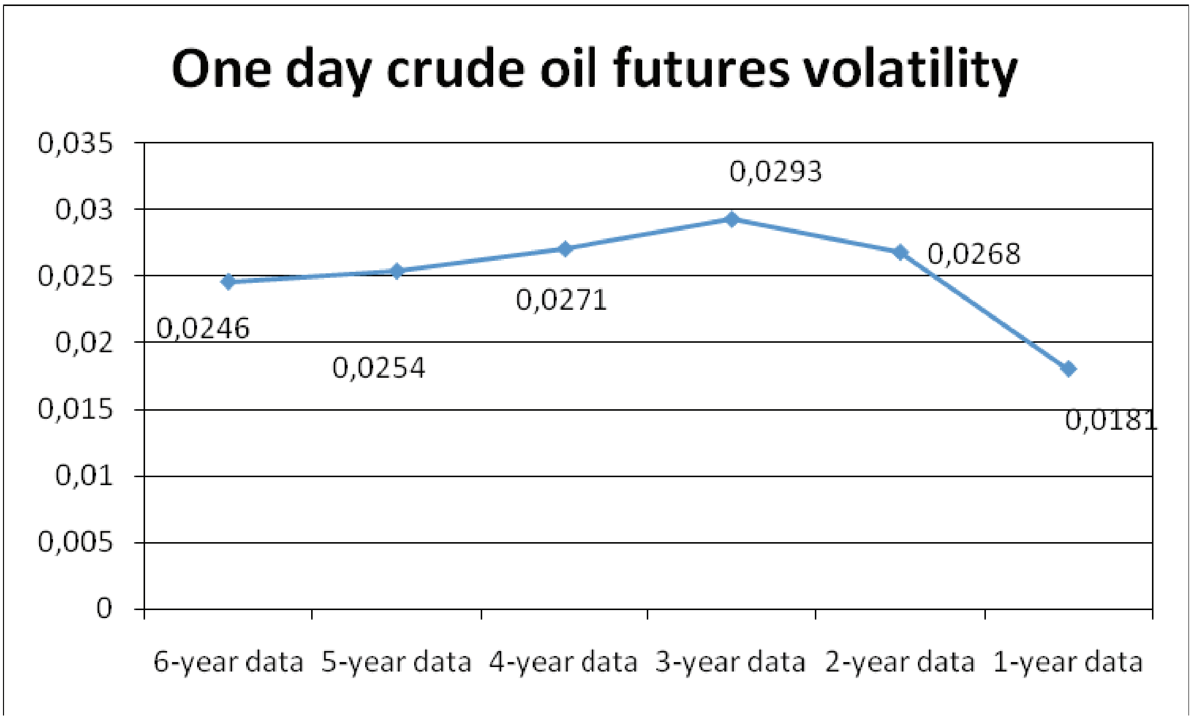


Source: Author's calculations.

The positive relation between the confidence level and value at risk is not a surprise, but it is worth emphasizing that the higher the confidence level is, the higher is the sensitivity of VaR changes to it. Thus, these are especially high levels of confidence at which the risk management is the most difficult and mistakes are most severe. Besides, the standard deviation level taken for the above calculations was counted for the six-year period starting from 2005 and ending with 2010. If it was calculated for a three, four or five-year period, the result would probably change.

Another measure that influences the VaR value is volatility. It is shown beneath that one-day volatility levels for different periods of time differ from each other. As it is depicted in chart 2, one-day volatility for six-year data is 2,46. It reaches its peak for three-year data and falls down up to one-year data.

Chart 2. Crude oil futures volatility for daily logarithmic rates of return for different time intervals.



Source: Author.

Case study II

Let's assume the same hedge fund which invests its assets of 100000000 USD in crude oil futures. Let's calculate value at risk for different confidence levels for a 10 – day holding period, provided that the daily volatility is counted for such periods as: 2006 – 2010, 2007 – 2010, 2008 – 2010, 2009 – 2010 and 2010.

The results of calculations are depicted in table 1 and show that the period of data taken for calculating the standard deviation influences VaR significantly for all analyzed confidence levels.

Table 1. Value at risk for investments in crude oil futures for different time intervals assumed for volatility calculations.

VaR confidence level	6-year period	5-year period	4-year period	3-year period	2-year period	1-year period
VaR 90%	10035172	10364223	11045585	11956483	10943497	7366353
VaR 91%	10424132	10765937	11473708	12419913	11367664	7651871
VaR 92%	10968676	11328337	12073081	13068714	11961497	8051596
VaR 93%	11513220	11890736	12672454	13717516	12555330	8451320
VaR 94%	12135557	12533479	13357451	14459003	13233996	8908148
VaR 95%	12835685	13256564	14128074	15293176	13997496	9422080
VaR 96%	13691397	14140335	15069945	16312721	14930663	10050219
VaR 97%	14702694	15184792	16183066	17517638	16033496	10792564
VaR 98%	16025158	16550619	17638686	19093299	17475662	11763324
VaR 99%	18125543	18719875	19950552	21595818	19766161	13305119
VaR 99,99%	30261010	31253354	33308004	36054821	33000158	22213267

Source: author's calculations.

Case study III

Under the same assumptions, let's calculate the VaR for Goldman Sachs CDS contracts and copper futures contracts. Results are depicted in tables 2 and 3. Charts 3 and 4 show volatility smiles for analyzed assets.

Table 2. Value at risk for investments in CDS contracts for Goldman Sachs for different time intervals assumed for volatility calculations.

VaR confidence level	6-year period	5-year period	4-year period	3-year period	2-year period	1-year period
VaR 90%	20766270	22257125	24644853	25134496	17108837	17879395
VaR 91%	21571165	23119804	25600080	26108701	17771970	18572394
VaR 92%	22698017	24327555	26937397	27472588	18700357	19542594
VaR 93%	23824868	25535306	28274715	28836476	19628743	20512794
VaR 94%	25112699	26915593	29803078	30395204	20689756	21621593
VaR 95%	26561509	28468416	31522486	32148774	21883396	22868993
VaR 96%	28332276	30366310	33623985	34292025	23342289	24393593
VaR 97%	30425001	32609276	36107575	36824959	25066436	26195392
VaR 98%	33161641	35542386	39355347	40137257	27321088	28551591
VaR 99%	37508070	40200854	44513572	45397965	30902008	32293790
VaR 99,99%	62620769	67116447	74316649	75793169	51591764	53915383

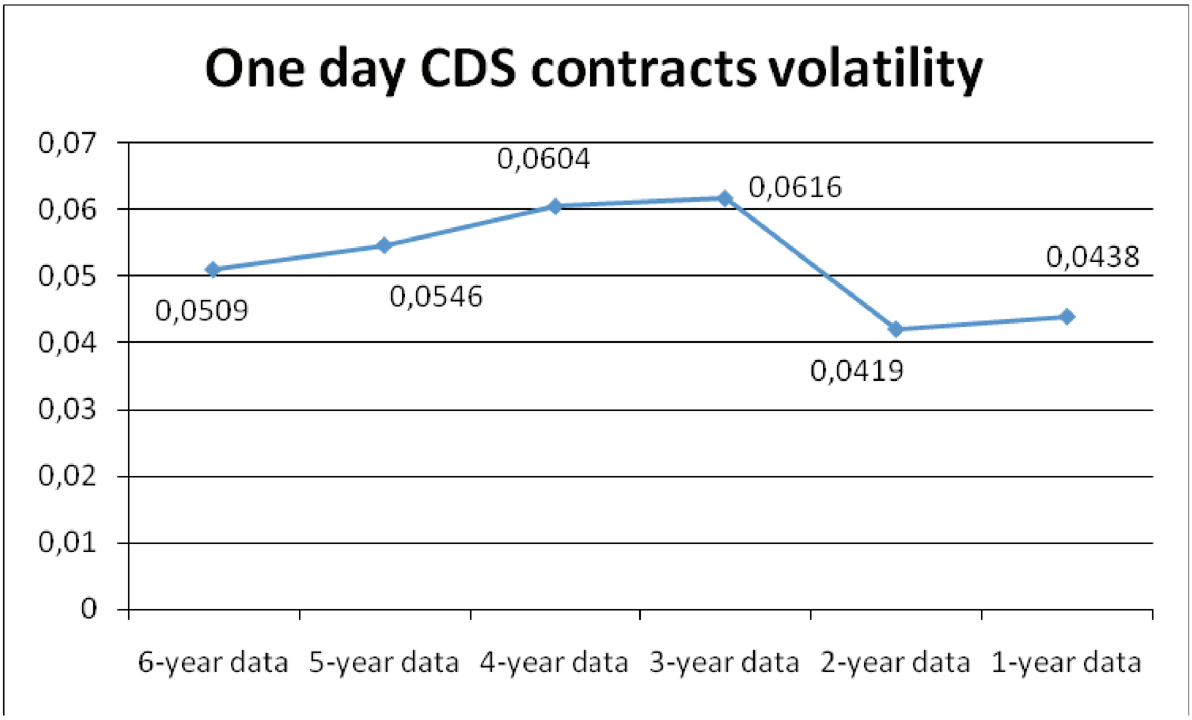
Source: Author's calculations.

Table 3. Value at risk for investments in copper futures for different time intervals assumed for volatility calculations.

VaR confidence level	6-year period	5-year period	4-year period	3-year period	2-year period	1-year period
VaR 90%	9358895	9871090	9862978	10307372	9097520	7220662
VaR 91%	9721643	10253690	10245264	10706882	9450137	7500532
VaR 92%	10229490	10789331	10780465	11266197	9943801	7892351
VaR 93%	10737337	11324971	11315665	11825512	10437465	8284170
VaR 94%	11317734	11937132	11927323	12464729	11001652	8731963
VaR 95%	11970680	12625813	12615437	13183847	11636362	9235730
VaR 96%	12768725	13467534	13456466	14062771	12412120	9851445
VaR 97%	13711870	14462295	14450410	15101498	13328924	10579109
VaR 98%	14945213	15763136	15750182	16459834	14527822	11530669
VaR 99%	16904051	17829178	17814527	18617191	16431954	13041970
VaR 99,99%	28221785	29766310	29741849	31081919	27433606	21773933

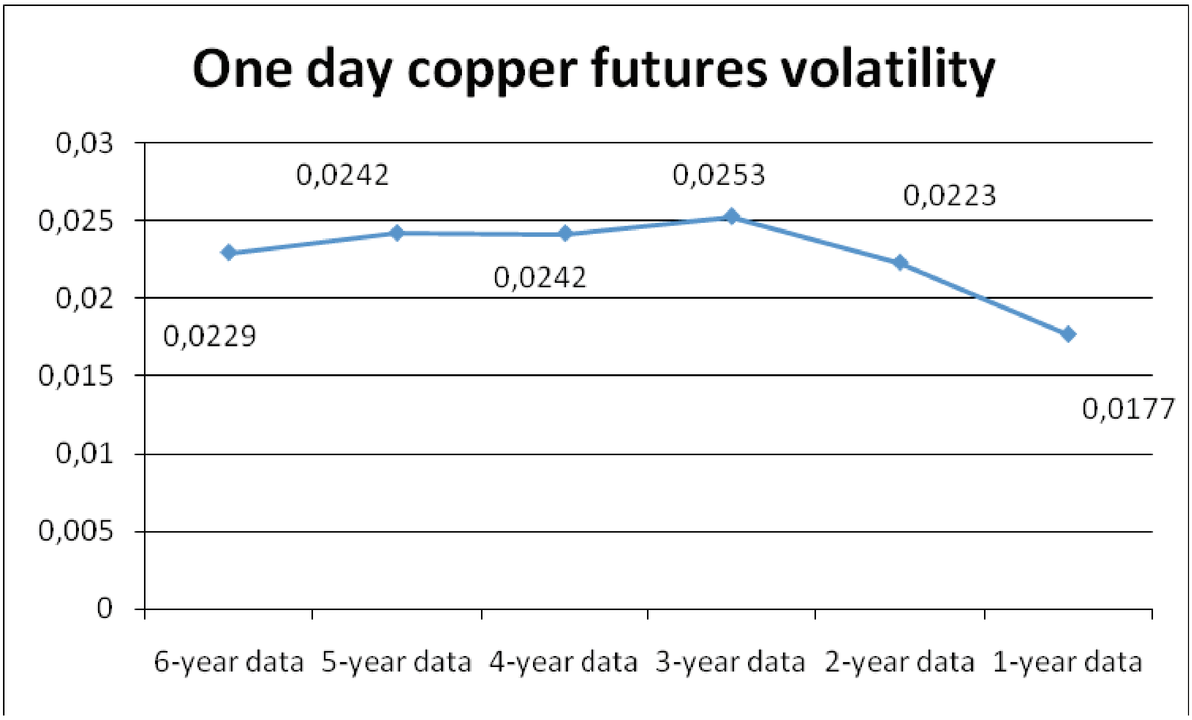
Source: Author's calculations.

Chart 3. CDS contracts for Goldman Sachs volatility for daily logarithmic rates of return for different time intervals.



Source: Author.

Chart 4. Copper futures volatility for daily logarithmic rates of return for different time intervals.



Source: Author.

The influence of different periods taken for volatility calculations on VaR level are summed up in table 4 which shows that fluctuations can even reach 32,69% (received for crude oil contracts).

Table 4. Percent changes of VaR.

Time interval	Fluctuations of VaR		
	Crude oil futures	CDS for Goldman Sachs	Copper futures
5-year period vs 6-year period	3,28%	7,18%	5,47%
4-year period vs 5-year period	6,57%	10,73%	-0,08%
3-year period vs 2-year period	8,25%	1,99%	4,50%
2-year period vs 3-year period	-8,47%	-31,93%	-11,74%
1-year period vs 2-year period	32,69%	4,50%	-20,63%

Source: Author's calculations.

If a hedge fund invests in many assets, these are not only problems with variance, skewness or kurtosis which are to be considered, but also correlation coefficients between investment assets are important. Correlation as a measure of dependence has some disadvantages. First of all, it measures linear dependence, which means that if it is low, it does not mean that dependence of examined variables is weak. Besides, models assume that it is unchangeable, whereas in fact it fluctuates. In perfect conditions, monitoring of all these measures in order to capture the real risk at a given point in time would have to be done continuously, which would cause costs of portfolios modifications to be extremely high. The correlation coefficient is given as:[5]

$$\rho_{ij} = \frac{\mathrm{cov}(r_i, r_j)}{\sigma_i \sigma_j}$$

where:

σ_i – standard deviation of rates of return on the i portfolio

σ_j – standard deviation of rates of return on the i portfolio

$\mathrm{cov}(r_i,r_j)$ – covariance between rates of return defined as:

$$\mathrm{Cov}(x,y) = \mathrm{cov}(y,x) = \frac{1}{n}\sum_{i=1}^{n}[(x_i - \bar{x})(y_i - \bar{y})]$$

where:

$\bar{x}$, $\bar{y}$ – avarage values of examined variables

In order to calculate VaR for a portfolio, one can use the following formula:[6]

$$\sigma_p = \sqrt{\rho_A^2 \times \sigma_A^2 + \rho_B^2 \times \sigma_B^2 + 2\rho_A \times \rho_B \times \rho_{AB} \times \sigma_A \times \sigma_B}$$

where:

σ_p – portfolio volatility

[5] G.W. Snedecor, W.G. Chochran, Statistical Methods, The Iowa State College Press, Ames, Iowa 1956, p. 168; M. Sobczyk, Statystyka. Podstawy teoretyczne, przykłady, zadania [Statistics. Theoretical foundation, examples, assignments], Wydawnictwo UMCS, Lublin 2000, p. 240 – 241; J. Jóźwiak, J. Podgórski, Statystyka od podstaw [Statistics from the rudiments], Polskie Wydawnictwo Ekonomiczne, Warszawa 2000, p. 48-49.

[6] P.Best, Wartość narażona na ryzyko. Obliczanie i wdrażanie modelu VaR [Value at risk. Calculating and implementing the VaR model], Dom Wydawniczy ABC, Oficyna Ekonomiczna, Kraków 2000, p. 36.

ρ_A, ρ_B – share of instruments A and B in the porfolio

σ_A, σ_B – A and B volatility

ρ_{AB} – correlation coefficient between A and B

In practice portfolio VaR is calculated with matrixes formulas:[7]

$$VaR_P = \sqrt{V \times C \times V^T}$$

Where:

VaR_p – portfolio VaR

V – row vector of VaR values for each individual position

C – correlation matrix

V^T – transposed matrix V

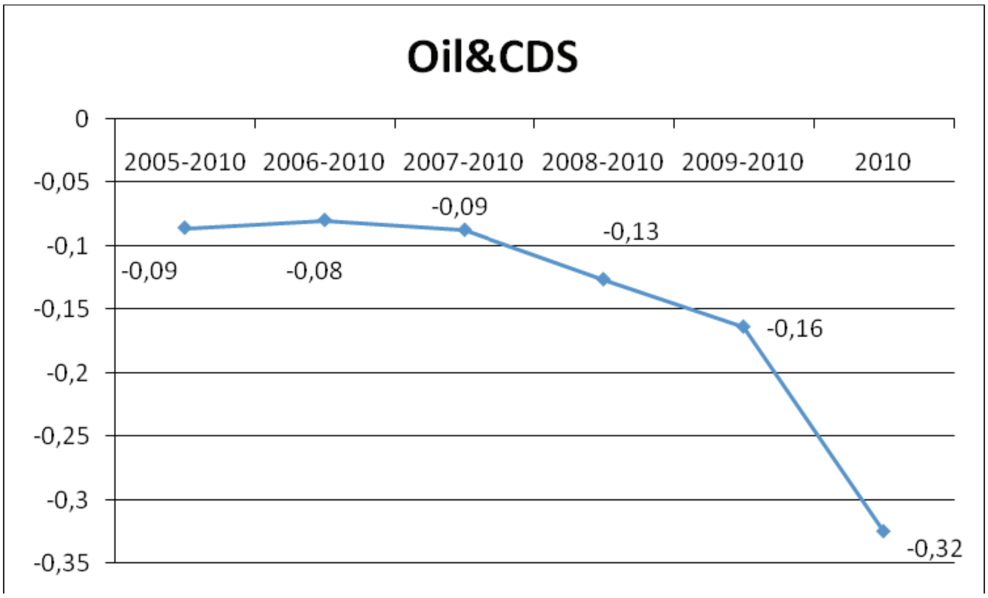
Case study IV

Let's calculate correlation coefficients for crude oil futures contracts, copper futures contracts and Goldman Sachs CDS contracts for different periods of time: 2005 – 2010, 2006 – 2010, 2007 – 2010, 2008 – 2010, 2009 – 2010.

Correlation coefficients for analyzed assets are depicted in charts 5,6,7. It is unquestionable that they are not unchangeable. For crude oil futures and CDS contracts, the shorter and more up to date the period of time, the lower the correlation is. For oil and copper, the shorter the period of time and the more recent the data, the higher the correlation is. And for CDS and copper, correlation goes down and up when the period of time shortens. Thus, apart from the fact that correlation changes in time, there are no clear trends for these changes.

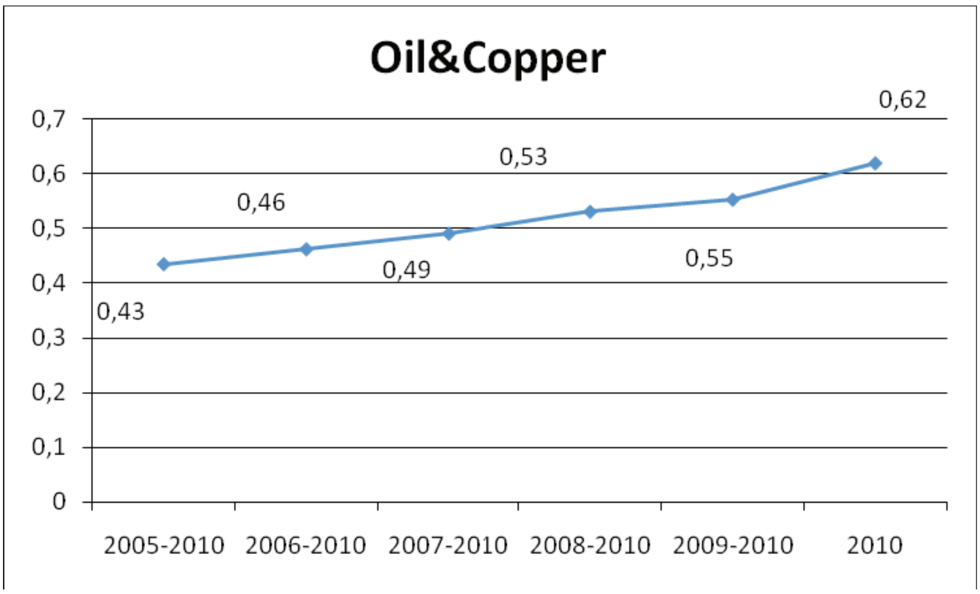
[7] Ibidem, p. 37.

Chart 5. Correlation coefficients for crude oil futures and Goldman Sachs CDS contracts for daily rates of return for different periods of time.



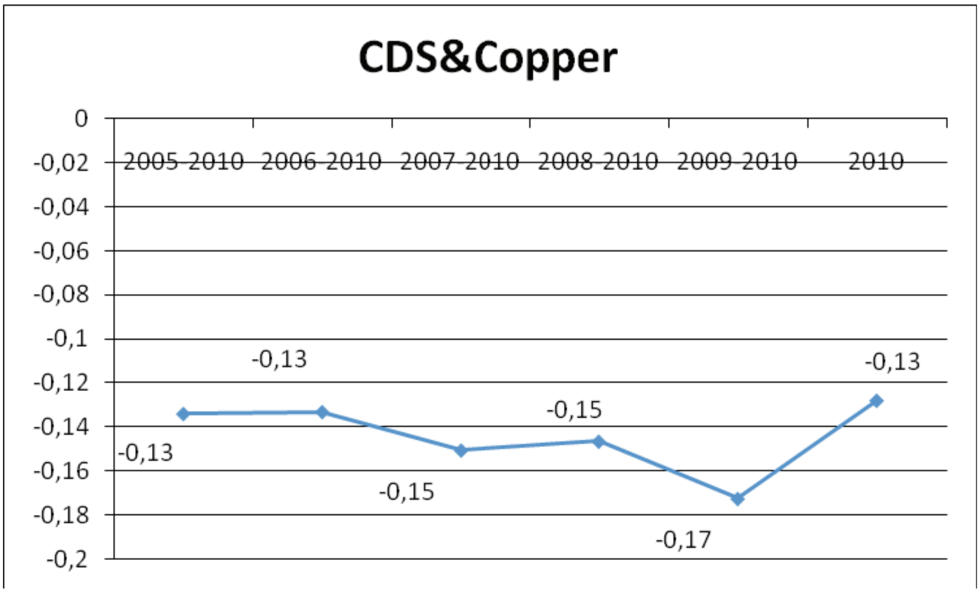
Source: Author's calculations.

Chart 6. Correlation coefficients for crude oil futures and copper futures contracts for daily rates of return for different periods of time.



Source: Author's calculations.

Chart 7. Correlation coefficients for copper futures and Goldman Sachs CDS contracts for daily rates of return for different periods of time.



Source: Author's calculations.

Case study V

Let's assume that a hedge fund constructs a portfolio made of three assets: crude oil futures contracts, copper futures contracts and Goldman Sachs CDS contracts. The value of each asset in the portfolio is equal to 1000000 USD. Assess the portfolio VaR for correlation coefficients for different periods, provided that the standard deviation does not change.

Table 5. Correlation coefficients of analyzed assets in 2005 – 2010.

Correlation coefficients	Crude oil futures	Goldman Sachs CDS contracts	Copper futures
Crude oil futures	1	-0,09	0,43
Goldman Sachs CDS contracts	-0,09	1	-0,13
Copper futures	0,43	-0,13	1

Source: Author.

Assuming that standard deviations computed above are given with 99% probability, row vectors are the following:

$1000000 \times 0{,}024617 = 24617$

$1000000 \times 0{,}050906 = 50906$

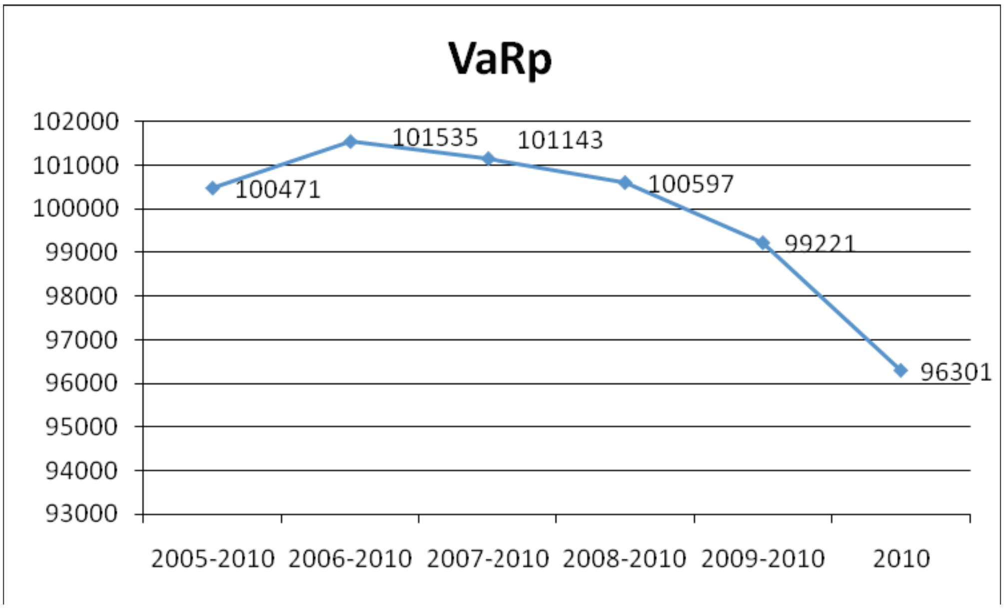
$1000000 \times 0{,}022942 = 22942$

Thus, the portfolio VaR_p is equal:

$$VaR_p = \sqrt{\left(\left\{[24617\ \ 50906\ \ 22942]\begin{pmatrix} 1 & -0{,}09 & 0{,}43 \\ -0{,}09 & 1 & -0{,}13 \\ 0{,}43 & -0{,}13 & 1 \end{pmatrix}\begin{bmatrix} 24617 \\ 50906 \\ 22942 \end{bmatrix}\right\}\right)} = 100471 \text{ USD}$$

Provided that standard deviations do not change, portfolio VaR for other correlation coefficients will change (see chart 8). It rises in the five-year period compared with a six-month period, next it falls down and is the lowest in a one-year period.

Chart 8. The influence of correlation coefficients calculated for different periods on portfolio VaR.

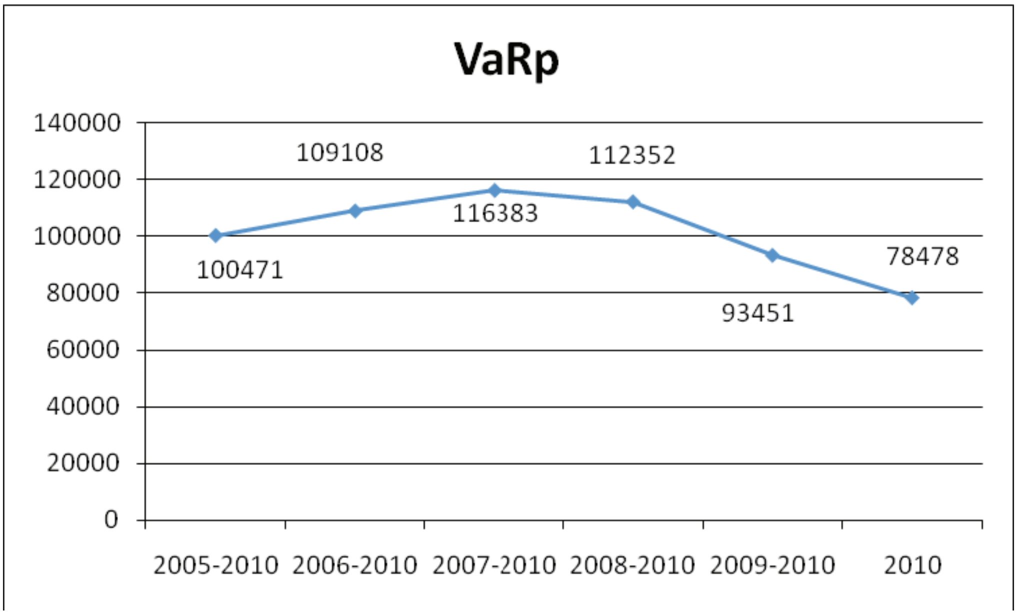


Source: author's calculations.

Case study VI

Let's calculate portfolio VaR for standard deviations and correlation coefficients taken for different time intervals. Results are depicted in chart 9.

Chart 9. Changes of portfolio VaR in relation to standard deviation and correlation coefficients fluctuations.



Source: Author's calculations.

If we take real values of both standard deviations and correlation coefficients, portfolio VaR fluctuates. First it starts to move up, reaching its peak in a four-year period and next it decreases up to a one-year period. If we look at percentage changes of VaR (see table 6), if correlation changes, they are not so substantial, however if we consider both correlation and volatility, they can be significant (from 3,46% to 16%).

Table 6. Percent changes of portfolio VaR.

Time interval taken for correlation coefficient and volatility calculations	Fluctuations of portfolio VaR	
	For changes of correlation only (standard deviation assumed to be unchangeable)	For changes of both correlation and standard deviation
5-year period vs 6-year period	1,06%	8,60%
4-year period vs 5-year period	-0,39%	6,67%
3-year period vs 2-year period	-0,54%	-3,46%
2-year period vs 3-year period	1,37%	-16,82%
1-year period vs 2-year period	2,94%	-16%

Source: Author's calculations.

CONCLUSIONS AND FINAL REMARKS FOR FURTHER STUDIES

- The introduction of different assumptions into risk models causes the improper risk level valuation by their users. It creates the need to develop risk management systems in financial institutions that consider the model risk.
- It has been known for many years that rates of return of the majority of assets are not normally distributed, as well as variance and correlation are changeable. However, any model is a simplification of reality. So, **these are not inadequate models but unsuitable people who do not take these simplifications into consideration and thus inadequate risk management systems that do not consider the model risk created by them**. The matter of model assumptions and model risk should be better emphasized in the process of teaching finance at universities. Models show the results which depend on our expectations of the market situation, which means that they incorporate our subjective appraisal. If risk of hedge fund investments is undervalued by banks, it will have consequences for the global financial market.
- It is not hedge funds and other institutions business to show what the risk generated by them really is, but banks should be interested in measuring and managing it properly. **If banks manage their exposures to hedge funds cautiously, the risk for the global financial market will be reduced.** The model risk should be incorporated into bank management systems. When they apply models, they should assume safety margins for the model risk.
- **Ratings given to hedge funds and other alternative investment vehicles by one international supervisory institution could reduce the problem of the moral hazard.** If there are a few rating agencies, the competition induces the moral hazard and increases the risk of the human factor. At the same time no one can expect that the systemic risk can be reduced only by the supervision of hedge funds. They cannot be controlled in full, which means that these are banks attitudes to transactions with hedge funds that should be verified, not only hedge funds themselves. It is widely acknowledged that stress tests can largely improve risk evaluation procedures. However, it is especially vital for transactions conducted with hedge funds because of their non-linearity and complexity. What's more, risk management of and in the hedge fund industry should be done with methods that include extreme risk measures and the asymmetry of financial instruments.

The credit risk transfer from hedge funds and other financial institutions into other parts of the financial market cannot be stopped but it can be managed by a better cooperation among banks and the integration of their risk management systems by supervisory institutions. In such a case warning systems could work better and contribute to the decrease of the systemic risk.

- And last but not least: it is not a matter of making models too complex, but of leaving some safety margins for the model risk. Models must be as simple as possible but not more.

References:

1. Best P., Wartość narażona na ryzyko. Obliczanie i wdrażanie modelu VaR [Value at risk. Calculating and implementing the VaR model], Dom Wydawniczy ABC, Kraków 2000.
2. Black F., M. Scholes, The pricing of options and corporate liabilities, Journal of Political Economy nr 81, May/June, 1973.
3. Brown S.J., W.N. Goetzmann, J.M. Park, Hedge Funds and the Asian Currency Crisis of 1997, 13 May 1998.
4. Chan N., M. Getmansky, S.M. Haas, A.W. Lo, Do Hedge Funds Increase Systemic Risk?, Economic Review, Federal Reserve Bank of Atlanta, Fourth Quarter 2006.
5. Choudhry M., The bond and money markets. Strategy, trading, analysis, Butterworth-Heinemann, Bath 2003.
6. Conine T.E., Jr., M.J. Tamarkin, On diversification given asymmetry in returns, Journal of Finance 36, no. 5, December 1981.
7. Credit Default Swaps and Counterparty Risk, European Central Bank, August 2009.
8. Crouchy M., D. Galai, R. Mark, Risk Management, McGraw-Hill, New York 2001.
9. S. Das, Credit Derivatives and Credit Linked Notes, John Wiley & Sons, Singapore 2000.
10. Duc F., Y. Schorderet, Market Risk Management for Hedge Funds. Foundations of the Style and Implicit Value-at-Risk, John Wiley & Sons, Ltd., Chichester 2008.
11. Elton E.J., M.J. Gruber, Risk reduction and portfolio size: An analythical solution, Journal of Business 50, October 1977.
12. Fama E.F., Risk, return and equilibrium: some clarifying comments, Journal of Finance, Vol. 23, No. 1, March 1968.
13. Fender I., J. Kiff, CDO rating methodology: Some thoughts on model risk and its implications, BIS Working Papers, Bank for International Settlements, Monetary and Economic Department, November 2004.

14. Guizot A., The Hedge Fund Compliance and Risk Management Guide, John Wiley & Sons, Inc., Hoboken 2007.
15. Hedge Fund Research, Inc., www.hedgefundresearch.com.
16. Hedge Fund Survey Overview, Capital Market Risk Advisors, inc., New York, 15 May 2000, www. cmra.com.
17. Jajuga K. (ed.), Zarządzanie ryzykiem [Risk management], PWN, Warsaw 2007.
18. Jajuga K, Krysiak Z.(ed.), Credit Risk of Mortgage Loans. Modelling and Management, Polish Bank Associaton, Warsaw 2005.
19. Jensen M.C., The performance of Mutual Funds in the period 1945-1964, Journal of Finance, 23, No. 2, 1968.
20. Jorion P., Value at Risk. The New Benchmark for Managing Financial Risk, McGraw-Hill, New York 2001.
21. Jóźwiak J., J. Podgórski, Statystyka od podstaw [Statistics from the rudiments], Polskie Wydawnictwo Ekonomiczne, Warszawa 2000.
22. Lai T., Portfolio selection with skewness, Review of Quantitative Finance and Accounting, No. 1, 1991,
23. Large EU Banks' Exposures to Hedge Funds, European Central Bank, November 2005.
24. Lhabitant F.S., Assessing market risk for hedge funds and hedge funds portfolio, Journal of Risk Finance, Spring 2001.
25. Lhabitant F.S., Handbook of hedge funds, John Wiley & Sons, Ltd., Chichester 2006.
26. Markowitz H., Portfolio selection, The Journal of Finance, Vol. 7, No. 1, March 1952.
27. McGuire P., E. Remolona, K. Tsatsaronis, Time – varying exposures and leverage in hedge funds, BIS Quarterly Review, March 2005.
28. OCC's Quarterly Report on Bank Trading and Derivatives Activities First Quarter 2008, Controller of the Currency, Administrator of National Banks, Washington DC 20219.
29. OCC's Quarterly Report on Bank Trading and Derivatives Activities First Quarter 2010, Controller of the Currency, Administrator of National Banks, Washington DC 20219.
30. Sharpe W.F., Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, Journal of Finance, Vol. 19, No. 3, September 1964.
31. Sharpe W.F., Mutual Fund Performance, Journal of Business, Vol. 39, No. 1, January 1966.

32. Sharpe W.F., Determining a fund's effective asset mix, Investment Management Review, No. 2(6), 1988.
33. Snedecor G.W., W.G. Chochran, Statistical Methods, The Iowa State College Press, Ames, Iowa 1956.
34. Sobczyk M., Statystyka. Podstawy teoretyczne, przykłady, zadania [Statistics. Theoretical foundation, examples, assignments], Wydawnictwo UMCS, Lublin 2000.
35. Tran V.Q., Evaluating Hedge Fund Performance, John Wiley & Sons, Inc., Hoboken 2006.
36. Treynor J.L., How to Rate Management of Investment Funds, Harvard Business Review, 43, No. 1, 1965.
37. Tufano P., Financial Innovation, a chapter in: G. Constantinides, M. Harris, R. Stulz, The Handbook of Economics of Finance, Harvard Business School, June 16, 2002.