RISK UNDERESTIMATION AS A CONSEQUENCE OF ASSUMPTIONS MADE IN VALUATION MODELS

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Abstract

The first part of the paper concentrates on the analysis of common risk models assumptions that are not fulfilled in practice. The most vital assumptions of the modern portfolio theory are discussed here and compared with reality to show that they do not come up to practice. The aim of the second part are empirical tests of some of these simplifications to justify opinions made in the previous section.

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1. CHOSEN SIMPLIFICATIONS OF VALUATION MODELS LEADING TO RISK UNDERESTIMATION

Model risk is defined as the special risk that arises when an institution uses mathematical models to value and hedge securities.¹ The author defines it as the probability of generating a loss that derives from the extent of model simplification in comparison to the real economy and from insufficient knowledge of the person who applies it. It is relatively lower for such instruments as stocks or bonds than for derivatives. The more complex the derivative is the higher the model risk is (see figure 3). One of the reasons is that the more advanced the instrument is, the more difficult it is to suit the model to it, as well as to understand this product and rules of its valuation together with its oversimplifications. This is why nowadays, it is more difficult to manage the model risk than it was 20 years ago, when nonstandard products were not as numerous as today. One can conclude that it is the

¹ M. Crouchy, D. Galai, R. Mark, Risk Management, McGraw-Hill, New York 2001, p. 579.

financial innovation that increases the model risk, defined by P. Tufano² as an ongoing process whereby private parties experiment to try to differentiate their products and services, responding to both sudden and gradual changes in the economy.

Figure 3. The relation between the complexity of the instrument and the model risk.



Source: Author.

The modern portfolio theory³ is based on the assumption that investors are in the setup of market equilibrium⁴, markets are perfect and efficient.⁵ Thus, treating investors as price takers who can't influence them has some far reaching consequences like not taking into consideration the speculation and the desire to influence the prices of cash markets in order to generate profits from derivatives markets. It could have been acceptable in the fifties or sixties when derivative markets were not so well-developed, however after Black and Scholes having done their model⁶, the financial world has changed. Paradoxically, it let the derivatives market grow and at the same time, it created the need for risk management on the market where more and more complex structures have appeared gradually. Derivatives changed the nature of the cash market. Although it is generally approved that these

² P. Tufano, Financial Innovation, a chapter in: G. Constantinides, M. Harris, R. Stulz, The Handbook of Economics of Finance, Harvard Business School, June 16, 2002, p. 2.

³ H. Markowitz, Portfolio selection, The Journal of Finance, Vol. 7, No. 1, March 1952, p. 77 – 91.

⁴ W.F. Sharpe, Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, Journal of Finance, Vol. 19, No. 3, September 1964, p. 425 – 442.

⁵ E.F. Fama, Risk, return and equilibrium: some clarifying comments, Journal of Finance, Vol. 23, No. 1, March 1968, p. 29 – 40.

⁶ F. Black, M. Scholes, The pricing of options and corporate liabilities, Journal of Political Economy nr 81, May/June, 1973, s. 637-659.

are derivatives whose prices depend on underlying market fluctuations, in fact there is also the impact of derivatives markets on cash markets that is often neglected. I mean here affecting prices of the cash market in order to generate profits from derivatives markets.

The most widely applied measure of risk-adjusted performance is the Sharpe ratio developed by Sharpe in 1966.⁷ It measures the excess return above the risk free rate compared with the unit of volatility given as the standard deviation of rates of return. Although it was proposed by Sharpe for mutual funds during the time when the hedge fund world was at its start (the first hedge fund is reported to have been created in 1954 by Alfred Winslow, however that was not the hedge fund of the type they function at present), it is widely used to compare hedge funds investment results nowadays, although they invest in asymmetric instruments. Furthermore, rates of return and volatilities must be estimated using a sample of returns, which creates another part of model risk – estimation risk. Besides, expected rates of return and variances of return are assumed to be constant and known to investors. In fact, they fluctuate together with prices on the market and there is no perfect information on the factors that influence them where the most unpredictable part of it is the human factor. Above all, the models do not take into account the credit risk that plays a significant role in practice.

Apart from the Sharpe ratio, two often met in practice risk-adjusted performance measures are the Treynor ratio⁸ and the Jensen Alpha⁹ that derive from the Capital Asset Pricing Model (abbreviated to CAPM) of William Sharpe¹⁰ which for example assumes that investors are risk-averse and do not take risk that can be diversified. Are really hedge funds as such if they invest borrowed capital and get commissions until the market goes in the right direction? When it starts to move in the opposite direction, they generate losses not for themselves, but for their customers. They have no motivation to be cautious being only rewarded for high rates

 ⁷ W.F. Sharpe, Mutual Fund Performance, Journal of Business, Vol. 39, No. 1, January 1966, p. 119
– 138.

⁸ J.L.Treynor, How to Rate Management of Investment Funds, Harvard Business Review, 43, No. 1, 1965, p. 63-75.

⁹ M.C., Jensen, The performance of Mutual Funds in the period 1945-1964, Journal of Finance, 23, No. 2, 1968, p. 389-416.

¹⁰ W.F. Sharpe, Capital Asset Prices: a Theory of Market Equilibrium Under Conditions of Risk, Journal of Finance, Vol. 19, No. 3, p. 425 – 442.

of return and not punished for high risk levels as their strategies are so complex and not revealed publicly that nobody can know what the real risk is.

The model suggested by Sharpe in 1988 treats the returns of an investment entity as a weighted average of portfolios or indices for the analyzed group of assets:¹¹

$$R_{t} = \sum_{i=1}^{n} \omega_{i} r_{i},$$

where:

 R_t – the return of an investment entity

 ω_{t} - the weight of asset i

 r_i – the return of asset i

This model is often met in practice as the linear regression model with the random factor. In the original Sharpe model there is an assumption that the manager cannot generate alpha, but it was later considered, allowing for possible excess returns:¹²

$$\sum_{R_t=\alpha+\sum_{i=1}^n \beta_i r_i + s_t}^n \beta_i r_i$$

where:

 α – measures the outperformance of the fund

 $\delta_{E} - error$

 β_{l} – the exposure to hedge fund strategies

However, the point is that it is linear, so it does not capture in the correct way the use of leverage or derivatives so often applied in the hedge fund industry. If one uses indices instead of assets this problem decreases, however another problem arises. Not all hedge funds are taken into consideration when indices are built. Thus, the question of representativeness appears.

Derivatives valuation models price these instruments in relation to the underlying asset market and assume that there are no arbitrage possibilities. It means that derivatives prices depend on underlying assets prices, however they

¹¹ W.F. Sharpe, Determining a fund's effective asset mix, Investment Management Review, No. 2(6), 1988, p. 59 – 69.

influence them at the same time, which together with the financial leverage applied, makes this dependence intensified. Besides, models are done on the assumption that there exists perfect liquidity of financial markets. Thus, the role of liquidity is marginalized and such events as the global financial crisis of 2007 – 2009 show that liquidity management is a crucial element of risk management.

Liquidity risk takes two forms:¹²

- Asset liquidity risk which arises when a transaction cannot be conducted at prevailing market prices due to the size of the position relative to normal trading lots.
- Funding liquidity risk which refers to the inability to meet payments obligations, which may force early liquidation, thus transforming "paper" losses into realized losses.

Both kinds of liquidity risk influence the market of both derivatives and cash market instruments, however they are not incorporated into pricing models. Asset liquidity risk is especially extreme on OTC exotic derivatives markets, as well as on emerging markets, whereas the funding liquidity risk is especially important for leveraged transactions conducted mostly by hedge funds.

Moreover, the value-at-risk model shows the worst case scenario, however during normal market conditions, at some level of probability and in a certain period of time. Normal market conditions mean that rates of return of assets follow a normal distribution and there is no risk of extreme events defined by K. Jajuga¹³ as those events that:

- Have low probability of appearance and
- Lead to big losses, and are thus called LFHS (Low Frequency, High Severity) events.

In fact, many distributions of rates of return of various assets are not normal and the effect of the so called "fat tails" appears. The probability of rates of return distant from the average is higher than the standard normal distribution shows.

¹² P. Jorion, Value at Risk. The New Benchmark for Managing Financial Risk, McGraw-Hill, New York 2001, p. 17 – 18.

¹³ K. Jajuga (ed.), Zarządzanie ryzykiem [Risk management], PWN, Warsaw 2007, p. 61.

There are three basic methods for VaR calculation and each of them is based on some untrue assumptions:¹⁴

- The variance covariance approach assuming that the distribution of rates of return is normal, correlation coefficients between risk factors do not change in time and the sensitivity of portfolio values to the change of risk factors does not change. Usually historical data are used for this method and the standard deviation is calculated.
- The historical simulation approach assuming that the distribution of rates of return does not change in time. Historical data are used to define the kind of the distribution and it is assumed to be the same in the future.
- The Monte Carlo simulation contrary to the previous two methods, it requires advanced computer software and is time – consuming. One creates a hypothetical model that describes rates of return fluctuations and generates many rates of return, which lets define the empirical distribution of rates of return. This approach is especially used when one needs to take into consideration such complex instruments as options, because the variance – covariance method does not consider gamma parameter, as well as exotic options that are priced with numerical methods.

It is alarming that many hedge funds apply basic risk management VaR analysis for their portfolios, however a minority of them deepens quantitative risk management practices to extreme value at risk, covariance analysis, and skewness framework. At the same time, the research has shown that many hedge funds exhibit significant skew and kurtosis.¹⁵ The main problems concerning the value-at-risk application are:

• The risk of the wrong VaR method application that does not take into consideration the appearance of extreme events

¹⁴ See f.ex. M. Choudhry, The bond and money markets. Strategy, trading, analysis, Butterworth-Heinemann, Bath 2003, s. 626 – 627.

¹⁵ E.J. Elton, M.J. Gruber, Risk reduction and portfolio size: An analytical solution, Journal of Business 50, October 1977, p. 415 – 437; T.E. Conine, Jr., M.J. Tamarkin, On diversification given asymmetry in returns, Journal of Finance 36, no. 5, December 1981, p. 1143 – 1155; T. Lai, Portfolio selection with skewness, Review of Quantitative Finance and Accounting, No. 1, 1991, p. 293 – 305, A. Guizot, The Hedge Fund Compliance and Risk Management Guide, John Wiley & Sons, Inc., Hoboken 2007, p. 113 – 114.

- The choice of the proper confidence level that can be changed on the grounds of the hedge fund policy or its procedures
- The proper choice of the time interval (the so called holding period) for which VaR should be calculated to be the adequate measure of risk
- VaR is not available for non-standard instruments that have low liquidity and thus historical price data for such structures are impossible to gather
- Skewness and kurtosis are not incorporated into the standard VaR model. The standard deviation used in the most common method of VaR calculation (variance – covariance) does not take them into consideration and does not measure the real level of risk.

The hedge fund survey done in 1999 by Capital Market Risk Advisors, Inc.¹⁶ shows that the variance – covariance approach was the most widely used method for calculating VaR. Although 1999 was a long time before the world global financial crisis of 2007 – 2009, it was shortly after the Asian crisis and the LTCM failure. It shows that no lessons were taken from it and still methods based on the normal distribution of rates of return were most commonly used by these institutions without doing any stress tests. Hewever, there is nothing bad in applying simplified models but the point is not to believe them in 100% and to take into consideration that extreme events may appear and to have enough of capital that will let manage the liquidity in case of them (the author calls it safety margins for the model risk).

2. EMPIRICAL VERIFICATION OF THE NORMAL DISTRIBUTION ASSUMPTION MADE FOR RATES OF RETURN FOR SELECTED HEDGE FUND INVESTMENT ASSETS

The first of tested assumptions is that rates of return of assets are normally distributed. It is shown with the example of three hedge funds investment assets that it does not have to be. Another assumption is that variance does not change in time, whereas in fact it not only fluctuates, but also significantly influences levels of value-at-risk. Besides, the assumption that correlation does not change in time is verified and it is concluded that its changes together with volatility fluctuations impact value-at-risk levels, which in turn means that value-at-risk cannot be treated as an absolute risk measure. It is often forgotten that both VaR and other models

¹⁶ Hedge Fund Survey Overview, Capital Market Risk Advisors, inc., New York, 15 May 2000, p. 1 – 13, www. cmra.com.

show risk level in standard market conditions or in conditions that one expects to turn out. In other words, one can calculate risk using these models, but they will only show the result which depends on our expectations of the market situation. And the point is how it will be understood and managed.

The author chose at random three investment assets which are widely known as those being the aim of hedge fund investments (crude oil futures contracts, CDS contracts for Goldman Sachs and copper futures contracts).

In order to check if rates of return of crude oil futures were distributed normally in 2005 - 2010, the author made the chi-square test. Chart 4 depicts the results of the test which lets reject the hypothesis of distribution normality with p = 0,00000 that the distribution is normal.

Chart 4. Normal distribution fitting by daily logarithmic rates of return of crude oil futures contracts in 2005 – 2010.



Source: Author.

Although the modern portfolio theory developed by Markowitz uses the standard deviation as a main measure of risk, rates of return of many assets do not follow the normal distribution. As chart 4 depicts, rates of return of crude oil futures contracts are dispersed around the mean more than the normal distribution assumes, however it is not the core of the matter. The biggest problem is that there are more rates of return far more from the mean than the normal distribution suggests (fat tails). Thus, the standard deviation is not a proper measure of risk for such distributions. Although it is rather low here (2,46%), risk is high because of excess kurtosis. Kurtosis is the fourth central moment of a distribution and its formal definition is:¹⁷

$$\frac{T(T+1)}{\text{Kurtosis}} = \frac{T(T+1)}{(T-1)(T-2)(T-3)} \sum_{t=1}^{T} \left(\frac{R_{t-1,t} - \overline{R}}{\sigma}\right)^{4} \frac{3(T-1)^{2}}{(T-2)(T-3)},$$

where:

- T the number of observations
- σ the standard deviation of rates of return
- $\overline{\mathbf{R}}$ arithmetic mean of rates of return

The normal distribution kurtosis is 3 and the value higher than this is considered excess kurtosis. For crude oil futures contracts it is 4,14 and shows another part of risk which is neglected in common valuation models.

Another risk measure omitted in the mentioned models is skewness. It is the third central moment of a distribution and measures the symmetry of a return distribution around the mean. Mathematically it is calculated as:¹⁸

Skewness =
$$\frac{T}{(T-1)(T-2)} \sum_{t=1}^{T} \left[\left(\frac{R_{t-1,t} - \overline{R}}{\sigma} \right) \right]_{3}$$

If the distribution is negatively skewed, it means that it is more probable that returns lower than average will be higher than returns higher than average. Even if two distributions have the same values of the standard deviation, the normal distribution with zero skew generates lower risk level than the non-normal distribution. Skewness of crude oil futures rates of return is close to zero and it is positive, which eliminates these problems (see table 1).

¹⁷ F.S. Lhabitant, Handbook of hedge funds, John Wiley & Sons, Ltd., Chichester 2006, p. 437. Note that some analysts do not subtract the second term from the kurtosis. As a result, when T is large, the threshold value for the normal distribution becomes 3 rather than 0.

¹⁸ F.S. Lhabitant, Handbook of hedge funds, John Wiley & Sons, Ltd., Chichester 2006, p. 436 – 437.

Item	Oil crude futures	CDS Goldman Sachs	Copper futures
Number of observations	1518	1505	1517
Mean	0,000510	0,001036	0,000704
Median	0,000813	0,00	0,001471
Minimum	-0,129589	-0,504155	-0,119575
Maximum	0,185866	0,571255	0,118506
Variance	0,000606	0,002591	0,000526
Standard deviation	0,024617	0,050906	0,022942
Standard error	0,000632	0,001312	0,000589
Skewness	0,006557	-0,178642	-0,150265
Standard error skewness	0,062807	0,063078	0,062828
Kurtosis	4,142480	26,36507	2,567517
Standard error kurtosis	0,125533	0,126072	0,125574

Table 1. Descriptive statistics of examined variables.

Source: Author's calculations.

Chart 5 shows that fat tails are present in the distribution of rates of return of CDS for Goldman Sachs. Besides, excess kurtosis appears (26,36), as well as negative skewness (-5,47). Thus, although the standard deviation is about 5,09%, risk is much higher than for the normal distribution. The chi-square test proves without any doubts that the hypothesis of the normal distribution of analyzed CDS rates of return can be rejected (with p = 0,00086 that the distribution is normal). The same conclusions can be drawn for chart 6 which suggests that the distribution of copper futures rates of return is not normal and data depicted in table 1 show that it is negatively skewed. The result of the chi-square test also confirms that copper futures rates of return are not normally distributed.



Chart 5. Normal distribution fitting by logarithmic daily rates of return of CDS contracts for Goldman Sachs in 2005 – 2010.

Source: Author.

Chart 6. Normal distribution fitting by logarithmic daily rates of return of copper futures contracts in 2005 - 2010.



Source: Author.

Additionally, chart 7 shows probability plots for analyzed variables which compare the model probability with the real probability. If the model is adequate, the probability plot lies close to the diagonal line. It can be seen that the fit is not so good and any deviations from linearity indicate some model failing.



Chart 7. Probability plots for examined variables in 2005 – 2010.

Source: Author.

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