THE TRAVELLING SALESMAN PROBLEM IN THE FUNCTION OF TRANSPORT NETWORK OPTIMALIZATION

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Abstract

The fundamental objective and purpose of this paper is to analyze the Travelling Salesman Problem (TSP) as a function of forming and optimizing transport networks. There are several software solutions for solving such problems, based on a heuristic algorithm. In practical application, the starting point consists of algorithms with solutions close to the optimum, or at least those with one optimal solution. Accordingly, the basic assumption of this paper is to use object modelling and programming in the spreadsheet interface (VBA for Excel), of which detailed analysis shows more than one optimal solution that could be used to create a flexible and adaptive transport network.

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Keywords: Travelling Salesman Problem (TSP), transport networks, optimization, object modelling, programming

1. Introduction

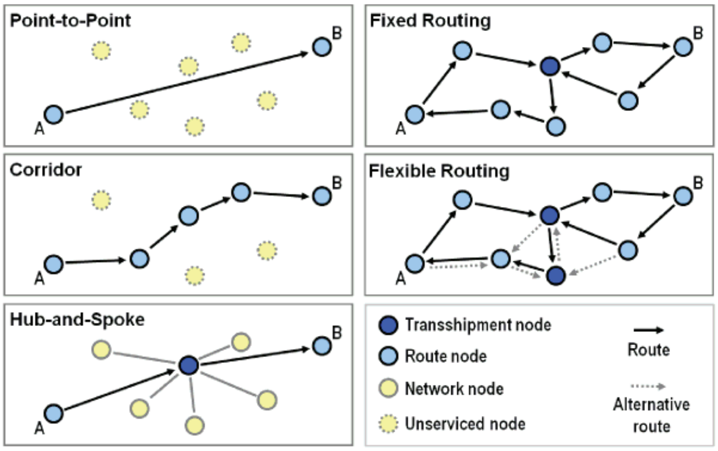
The Travelling Salesman Problem (TSP) is the problem of discrete and combinatory optimization. It belongs to the NP-difficulty group of problems, its complexity is O (n!). Mathematical problems similar to the Travelling Salesman Problem were first considered by Euler, who wanted to know how the jumper on the chessboard would visit all the 64 fields only once. In the early 20th ct, mathematicians William Rowan Hamilton and Thomas Kirkman discussed the problems which come down to the Travelling Salesman Problem, and its early general form appears in the 30's of the 20th ct. The term 'salesman' was first used in 1932.

The majority of existing software solutions allows calculation and insight into one optimal solution. Using visual and object methods in programming and modelling to form an algorithm of detailed search criteria can simulate models with more than one optimal solution for small scale patterns, with clear interpretation of the results, not only those in optimal value, but also those of approximately equal values and their deviation from the optimum. Finding a large number of optimal transport relations allows greater flexibility in making a multiobjecitive selection of optimal transport relation, especially over different periods of time. In this paper, the basic criterion for selection of optimal transport relation is the distance between cities (trade-transport centres). In cases of the same or similar distance, there is a possibility of dynamic selection of multiple transport relations for different periods of time, so, from the perspective of other relevant criteria, there can be one optimal relation for a certain period of time, and another optimal relation for other periods.

2. Theoretical Background and Problem of Research

Logistics have distinct geographical dimension, expressed in terms of flows, nodes and networks within the supply chain (Rodrigue et al; 2006, 161). The spatial structure of contemporary transport networks is the outcome of the spatial stucture of distribution. The networks setting leads to a shift towards larger distribution centres, and often serves as significant transnational flux. However, there is no demise of national or regional distribution centres, with some goods still requiring a three-tier distribution system (regional, national and international distribution centres). Figure 1 illustrates five main network strategies.

Figure 1. Freight distribution and transport network strategies



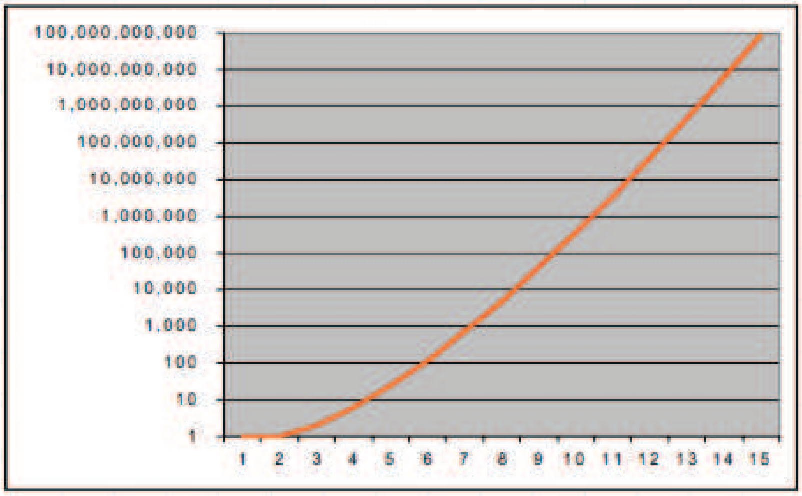
Source: Adapted from Woxenius, J. (2002) Conceptual Modelling of an Intermodal Express Transport System, International Congress on Freight Transport Automation and Multimodality: Delft, The Netherlands.

The following deals with distribution strategy in short (Rodrigue et al, 2006, 163-164). **Point-to-point** distribution is common with completion of specialized and specific one-time orders, which often results with less-than-full-load as well as empty return problems. Logistical requirements of such a structure are minimal, but at the expense of efficiency. **Corridor** structures of distribution often link high density agglomerations with services such as the landbridge, where container trains link seaboards. Traffic along the corridor can be loaded or unloaded at local / regional distribution centers, acting as sub-hubs in this distribution system. **Hub-and-spoke** networks have mainly emerged with air freight distribution and with high throughput distribution centers favored by parcel services. Such a structure is made possible only if the hub has the capacity to handle large amounts of time-sensitive consignments. The logistical requirements of a hub-and-spoke structure are consequently extensive as efficiency mainly derives at the hub's terminal. Commonly, a major distribution center located at the hub will have privileged access to a terminal. **Routing** networks tend to use circular configurations where freight can be transshipped from one route to the other at specific hubs. Pendulum networks characterizing many container shipping services are relevant examples of relatively fixed routing distribution networks. Achieving flexible routing is a complex network strategy requiring a high level of logistical integration as routes and hubs are shifting depending on anticipated variations of the integrated freight transport demand.

If we observe distribution of goods, it can be considered that its efficiency is proportional to degree of the transport network construction, and strategic distribution planning should be based on the optimal movement drove of a transport network. Strategic planning requires managers to understand types of factors in an environment (Certo & Certo, 2008, 181), and accordingly, to build a flexible and adaptive transport network. Transport network flexibility and adaptability can be achieved by optimizing routes for vehicles moving from one place of departure to specific, more than one transport destination. The most important operational decision related to transportation in a supply chain is routing and scheduling of delivers (Chopra, Meindl, 2001, 284).

The Travelling Salesman Problem (TSP) can be formulated as follows: to choose a pathway optimal by the given criterion. In this, optimality criterion is usually the minimal distance between towns or minimal travel expenses. Travelling salesman should visit a certain number of towns and return to the place of departure, so that they visit each town only once. The Travelling Salesman Problem (TSP) is one of the most studied problems in management science. Optimal approaches to solving Travelling Salesman Problems are based on mathematical programming. But in reality, most TSP problems are not solved optimally. When the problem is so large that an optimal solution is impossible to obtain, or when approximate solutions are good enough, heuristics are applied. Two commonly used heuristics for the Travelling Salesman Problem are the nearest neighbour procedure and the Clark and Wright savings heuristic (Heizer & Render, 2004, T 5-5). The complexity of TSP is vividly demonstrated in Chart 1.

Chart 1: Number of alternative routes with n towns



Source: Caplice, C. Logistic Systems, MIT Center for Transportation & Logistics

The Travelling Salesman Problem (TSP) from the perspective of combinatorial optimization can be formulated as follows: given a set of n towns C and travel distances dij from town i to town j. Starting from a given town, it is necessary to visit each town only once, so that the total length of travel distance is minimal. In accordance with the above definition, derive the following formulas and terms: 1) lengths of transport distances (distances between towns) are symmetrical: (1) dij = dj, 2) input variable is defined as a finite set of towns C = (c_1 … c_n), and the distance matrix is defined in terms of d(ci, cj), which indicates the distance between town ci and town cj for each pair i,j. Since the distance matrix is symmetrical, the following formula applies cij=cji; 3) permutations are calculated as resultant variables, that is all permuted relations to be gained by a given number of towns. Permutations p(1),....,p(n) from the set 1,....,n are calculated and compared so that the sum of the formula is minimal.

Consequently, the Travelling Salesman Problem can be expressed by the formula:

$$\sum_{1 \le i \le n-1} d(c_{\pi(i)}, c_{\pi(i+1)}) + d(c_{\pi(n)}, c_{\pi(1)}) \tag{1}$$

The formula represents the sum of lengths of relations starting in town cp(i), i=1, so that each town is visited in a particular sequence (cp(i), cp(i+1)), and finally to return to the town of initial departure cp(1). This algorithm calculates the length for all possible relations and finds the relation with the smallest length. Also, the number of possible relations is factorial of n number of towns, that is, the number of permutations of n elements.

Object program for the algorithm of detailed search in the spreadsheet interface explores and finds all relations with the minimal value achieved. Also, the program can explore and find relations with values close to optimal (minimal) value with predefined minimal deviation. Crucial factor for structuring a transport network with transportation at minimal cost, maximal profits and minimal time is the use of relevant information technologies and computer applications that allow the calculation of the optimal connectivity of nodes (towns) and scheduling of transport relations. Despite this, the level of using computer-supported methods for optimization of transport networks in Croatian companies is significantly lower compared to needs and opportunities provided by natural, human and technological resources.

3. Research Results and Discussion

This section discusses the Travelling Salesman Problem in analysis of a transport network on a practical example. Here, the connection between towns is given: Croatia: Rijeka (RI) and Zagreb (ZG); Italy: Trieste (TR) and Udine (UD); Slovenia: Ljubljana (LJ), Celje (CE) and Maribor (MB); and Austria: Klagenfurt (KL) and Graz (GZ). This example is chosen because it is expected that by joining the EU the Port of Rijeka as a significant refractive traffic point will become an even more important source of trade flows in the regional transport network. Relations between towns are provided in Table 1.

Table 1: Relations between towns within regional transport network

1	142	160	331	266	220	147	220	74	**1**	**RI**
142	1	143	195	130	78	159	88	93	**2**	**LJ**
160	143	1	189	121	152	291	224	224	**3**	**ZG**
331	195	189	1	70	123	301	136	288	**4**	**GR**
266	130	121	70	1	55	287	131	220	**5**	**MB**
220	78	152	123	55	1	238	161	170	**6**	**CE**
147	159	291	301	287	238	1	161	73	**7**	**UD**
220	88	224	136	131	161	161	1	170	**8**	**KL**
74	93	224	288	220	170	73	170	1	**9**	**TR**
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**		
RI	**LJ**	**ZG**	**GR**	**MB**	**CE**	**UD**	**KL**	**TR**		

Source: By Author, according to Google Maps.

Highways are used for connecting towns. When distance and estimated time are smaller on the motorway, compared to a highway which connects the same towns in question, it is recommended to use the motorway. The information from Table 2 shows that by object modelling and interface spreadsheet programming using detailed search four optimal relations are obtained, plus two more relations with 1% deviation.

Table 2: Optimal solutions of movement drove on the regional transport network

	A	B	C	D	E	F	G	H	I	J	K
1	1	9	7	8	2	6	4	5	3	**948**	**0,0%**
2	1	3	5	4	6	2	8	7	9	**948**	**0,0%**
3	1	9	7	8	2	6	5	4	3	**948**	**0,0%**
4	1	3	4	5	6	2	8	7	9	**948**	**0,0%**
5	1	9	7	8	4	5	6	2	3	**950**	**0,2%**
6	1	3	2	6	5	4	8	7	9	**950**	**0,2%**
7	1	9	7	2	8	4	5	6	3	**967**	**2,0%**
8	1	3	6	5	4	8	2	7	9	**967**	**2,0%**
9	1	7	9	2	8	4	5	6	3	**974**	**2,7%**
10	1	3	6	5	4	8	2	9	7	**974**	**2,7%**
40319	1	6	7	5	9	4	2	3	8	**2035**	**114,7%**
40320	1	8	3	2	4	9	5	7	6	**2035**	**114,7%**

Source: Author's calculations

The information from Table 2 show four optimal solutions for movement drove on the regional transport network:

RI – TR – UD – KL – LJ – CE – GR – MB – ZG – RI

RI – ZG – MB – GR – CE – LJ – KL – UD – TR – RI

RI – TR – UD – KL – LJ – CE – MB – GR – ZG – RI

RI – ZG – GR – MB – CE – LJ – KL – UD – TR – RI

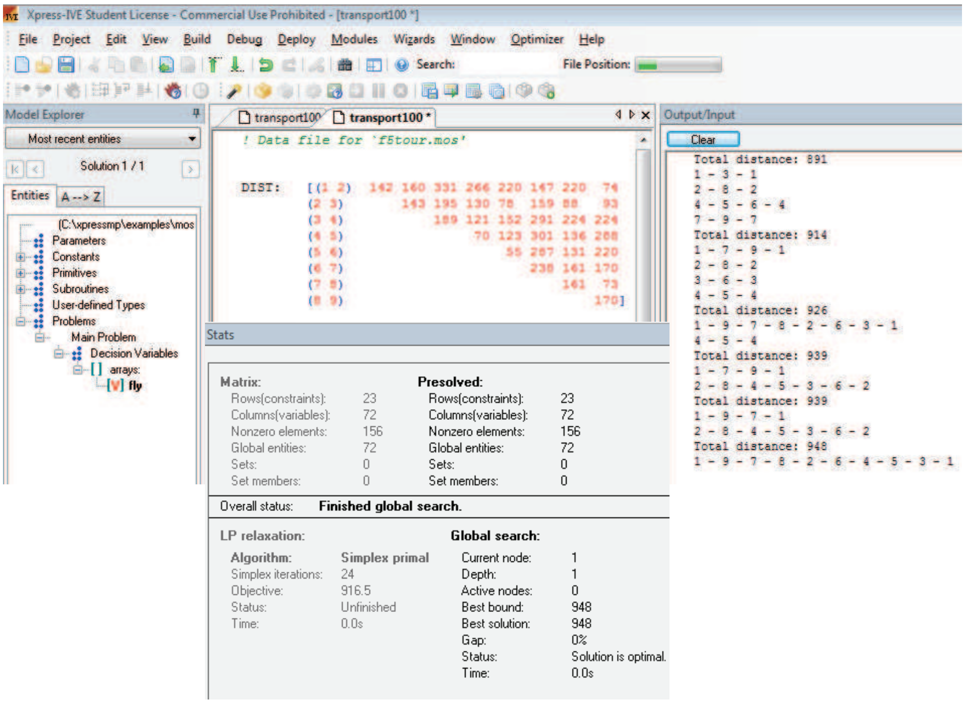
The information from Table 2 show two suboptimal solutions for movement drove on the regional transport network within 1% deviation:

RI – TR – UD – KL – GR – MB – CE – LJ – ZG – RI

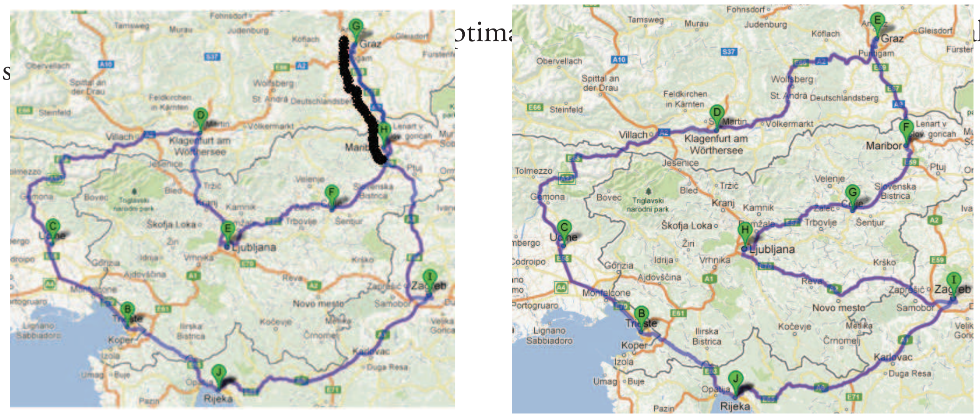
RI – ZG – LJ – CE – MB – GR – KL – UD – TR – RI

Accuracy of the above data is verified by comparing solutions obtained by using Xpress, a program language for mathematical modelling (cf. Figure 2).

Figure 2: Optimal solution of movement drove on the regional transport network by use of Xpress



What appears to be interesting is that all optimal solutions include redundant relation (cf. Map 1a).



Source: Author's calculations

Redundant relation (GR-MB) means any gap in the context of responsive movement, and hence, of the dual movement on the same route. Formally, mathematically speaking, the optimal relational line, in this example as well, passes exactly once through each node. Realistically, the optimal relation passes through Maribor twice.

For suboptimal solutions (cf. map 1b) within acceptable deviation, there is no redundant relation, and the total movement drove is increased by only 2 km.

4. Conclusion

Object modelling and programming in spreadsheet interface (VBA for Excel) using the method of detailed search, results with a larger number of optimal alternative routes on the transport network that are examined in solving the Travelling Salesman Problem. Finding a larger number of optimal transport relations enables management to achieve greater flexibility and adaptivity of companies and also, faster and easier decision making. Thus managers can consider different optimal alternative solutions and choose the most beneficial solution from the perspective of various relevant criteria. Empirical research confirmed that even suboptimal solutions (within 1% deviation) contribute to effective problem solving.

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