

TRANSPORT NETWORK OPTIMIZATION BASED ON FINDING OPTIMAL AND SUBOPTIMAL SOLUTIONS ON THE EXAMPLE OF THE RIJEKA URBAN AGGLOMERATION

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Abstract

The paper investigates the modeling and optimization of the transport network in finding optimal and suboptimal solutions on the example of the Rijeka Urban Agglomeration with the aim of achieving and improving business results. The urban agglomeration of Rijeka consists of 14 cities and municipalities, which are also called transport hubs, and are connected by routes in the transport network. By modeling and optimizing the transport network of the Urban Agglomeration, Rijeka should provide optimal service to cities, municipalities and their distribution centers in the area of the agglomeration.

The basic criterion for choosing optimal transport routes is the distance between cities (transport nodes). If the distance is the same or similar, dynamic selection can determine multiple transport routes in different periods can be obtained by dynamic selection, so from the point of view of other relevant criteria, one route can be optimal in one period, and another route can be optimal in another period. Based on the exhaustive search algorithm used to solve the traveling salesman problem (TSP), a visual model of the optimization of the transport network of the Urban Agglomeration of Rijeka can be created using the Visual Basic program in the Excel spreadsheet interface.

In the example of optimization of the transport network of the Rijeka Urban Agglomeration, the optimization factors are the minimum length of the transport route, the shortest time at minimum cost, and the maximum utilization of transport capacity. By considering optimal and suboptimal solutions within a given deviation interval, it is possible to break down and analyze the synergy of all relevant factors

that determine the best (optimal or suboptimal) solution in different situations in order to achieve the minimum length of the transport. The optimization of the transport network based on finding optimal and suboptimal solutions has significant impact on the efficient and flexible optimization of transport network, which enables the choice between alternative transport routes in different situations. The paper investigates the impact of optimization of the transport network based on the calculation of optimal and suboptimal solutions on business effects in the logistics and transport system.

Key words: transport network optimization, suboptimal solutions, alternative roads, business effects, Rijeka Urban Agglomeration,

1. INTRODUCTION

Due to the large increases in the number of cities in the world, mobility between cities has become difficult because there are many different roads to reach the same city with different travel costs (Ameen, Sleit, & Al-Sharaeh, 2018), where there are several places all directly connected by different long roads, and the passenger wants to make the shortest trip. Some algorithms can be used to guide people using any of the transportation or movement methods (walking, train, car, or bus) to their destination by the shortest route (Zhen & Noon, 1996.).

In this paper, we present the application of the Traveling- Salesman problem (TSP) and an exhaustive search algorithm on the example of the transport network of the Urban agglomeration of Rijeka. The rational use of space and other resources for transport purposes is a very important factor in the planning and design of the transport network of an urban agglomeration (Smojver, Baričević & Schiozzi, 2018). The development of urban agglomeration has led to a large number of potential solutions for the design of a flexible and optimal transport network.

Object modeling and programming in a spreadsheet interface (VBA for Excel) using the exhaustive search algorithm achieves a greater number of optimal alternative routes on the transport network that are examined in the transport network optimization. Finding a greater number of optimal and suboptimal transport routes enables the management to achieve greater flexibility and adaptability of the company, as well as faster and easier decision-making. Thus, managers considering various optimal alternative solutions can choose the most useful solution from the point of view of various relevant criteria (Vukmirović, Pupovac, 2013). The Exhaustive Search algorithm used in the modeling of the transport network of the cities and municipalities of Urban agglomeration Rijeka makes it possible to find more optimal and suboptimal solutions, which significantly affects the efficient and flexible optimization and modeling of the transport network, enabling a choice between alternative transport routes in different situations, time and cost savings, better utilization of transport capacity, creating added value and achieving higher profits.

Identifying multiple optimal transportation routes (an optimal solution and suboptimal solutions) allows for greater flexibility in optimizing and modeling the transportation network with the goal of computing several different transportation routes that meet different criteria, such as different transportation needs in different

time periods. Among the important decisions related to the optimization of the transport network using the example of the Rijeka agglomeration is the calculation and representation of several alternative transport routes based on suboptimal solutions calculated together with an optimal solution (Vukmirović, Čapko & Babić, 2019a).

Visual and object-oriented methods of modelling and programming and Exhaustive Search Algorithm allows us to solve and visualize the Travelling Salesman Problem, in the way to identify multiple optimal solutions with a clear interpretation of the results not only of the optimal value but also at approximately equal values and their deviations from the optimal value. Given that Urban Agglomeration Rijeka covers 14 cities and municipalities, Exhaustive Search Algorithm is suitable for designing the transport network. In making the program based on the Exhaustive Search Algorithm have been used Visual Basic for Application in Excel spreadsheet interface. The Exhaustive Search Algorithm, with the calculation of the optimal relation, also allows the calculation of suboptimal relations whose values within an acceptable deviation from the optimal value.

In investigating the problem above, the following scientific hypothesis has been proposed: Visual and object-oriented methods of modelling and programming in the spreadsheet interface, on the base of Exhaustive Search Algorithm for solving Traveling Salesman Problem, enable transport network optimization that identify multiple optimal and suboptimal solutions and enable significant influence on the business effects of transport in Rijeka Urban Agglomeration: reduction of transport time and costs, greater safety of transport within the time schedules and better utilization of vehicle capacity.

The paper is divided into seven main chapters:

The **Introduction** is the first chapter and introduces the reader to the research problem of the thesis, the hypothesis and the overall structure of the thesis.

The second chapter entitled **Traveling salesman problem (TSP), Exhaustive Search Algorithm and Object programming in the spreadsheet interface** consists of previous research and presentation of the exhaustive search algorithm in solving TSP in a spreadsheet interface, considering the example of the base of a square pyramid in the calculation and finding a large number of optimal and suboptimal solutions of the transport network, as well as the visualization and transparency of the solutions in the spreadsheet interface.

The third chapter entitled **Urban agglomeration of Rijeka and transportation network** considers the significance of the transport network in connecting the City of Rijeka, the Port of Rijeka and the cities and municipalities of the Urban Agglomeration of Rijeka

In the fourth chapter, titled **Conceptual Model of Transport Network Optimization** on the Example of the Urban Agglomeration of Rijeka, the transport network is divided into two segments from the point of view of identifying repeated transport routes and finding optimal and suboptimal solutions.

The fifth chapter, titled **Exhaustive Search Algorithm Approach to Finding the Suboptimal Solutions of the Transportation Network**, discusses and describes the use and importance of the Exhaustive Search Algorithm, based on object-oriented

modeling and programming methods in finding and visualizing optimal and suboptimal solutions for the selected transportation network segment.

In the sixth chapter, titled the **Significance of identifying and finding more optimal and suboptimal solutions of the transport network to the business effects**, there have been analysed and explained influence on the business effects of transport in Rijeka Urban Agglomeration: reduction of transport time and costs, greater safety of transport within the time schedules and better utilization of vehicle capacity.

The last chapter is the **Conclusion**, which includes a summary statement and presents a synthesis of the key points and the main research results.

2. TRAVELING SALESMAN PROBLEM, EXHAUSTIVE SEARCH ALGORITHM AND OBJECT PROGRAMMING IN THE SPREADSHEET INTERFACE

The Travelling Salesman Problem (TSP) is an optimization problem used to find the shortest path to travel through the given number of cities. Travelling salesman problem states that given a number of cities N and the distance between the cities, the traveler has to travel through all the given cities exactly once and return to the same city from where he started and also the length of the path is minimized (Rao, Hegde, 2015).

The Travelling Salesman Problem (TSP) can be formulated as follows: to choose a pathway optimal by the given criterion. In this, optimality criterion is usually the minimal distance between towns or minimal travel expenses. Travelling salesman should visit a certain number of towns and return to the place of departure, so that they visit each town only ... The travelling salesman problem can be classified as Symmetric Travelling Salesman Problem (STSP), and Asymmetric Travelling Salesman Problem (ATSP). In STSP the distance between two cities is same in both the directions. In ATSP the distance between two cities is not same in both directions (Rao, Hegde, 2015).

In terms of combinatorial optimization, the Travelling Salesman Problem (TSP) can be formulated in the following way: Given a list of n cities C and distance d_{ij} from city i to city j ; TSP, is to find the best possible way of visiting all the cities by visiting each city only once finding minimum total travel distance. In analogy to the above definition, the following formulations are valid: 1) Travel distance or distance between cities is symmetric: $d_{ij} = d_{ji}$ (1) or asymmetric $d_{ij} \neq d_{ji}$ (2); 2) Final list of cities is defined as incoming variable by the formula $C = (c_1 \dots c_n)$, while distance matrix containing distance between city c_i and city c_j for each pair i, j is defined by $d(c_i, c_j)$; 3) Permutations or in other words all permuted relations that can be achieved for a given number of cities are computed as resulting variables. Permutations $p(1), \dots, p(n)$ in the list $1, \dots, n$ are calculated and compared to give the minimum sum (Abdoun, Abouchabaka, Tajani 2012), (Vukmirović, Pupovac, 2013).

Exhaustive search algorithm, also known as brute force search, is a very general problem-solving technique. This algorithm calculates the length for all possible relations and finds the relation with the smallest length. Also, the number of possible relations is factorial of n number of towns, that is, the number of permutations of n

elements. In the Travelling Salesman Problem (TSP), every tour corresponds to a permutation of the cities. In a permutation problem every feasible solution can be specified as a total ordering of an underlying ground set (Fomin, F.V., Kratsch, D., 2010). The Exhaustive Search Algorithm enumerating all possible candidates for the solution (permutations) and checking whether each candidate satisfies the problem's statement. It is considered as approach to apply and is useful for solving small-size instances of a problem.

The majority of existing software solutions allows calculation and insight into one optimal solution. Using visual and object methods in programming and modelling to form an algorithm of detailed search criteria can simulate models with more than one optimal solution for small scale patterns, with clear interpretation of the results, not only those in optimal value, but also those of approximately equal values and their deviation from the optimum. Finding a large number of optimal transport relations allows greater flexibility in making a multiobjective selection of optimal transport relation, especially over different periods of time. In this paper, the basic criterion for selection of optimal transport relation is the distance between cities (trade-transport centres). In cases of the same or similar distance, there is a possibility of dynamic selection of multiple transport relations for different periods of time, so, from the perspective of other relevant criteria, there can be one optimal relation for a certain period of time, and another optimal relation for other periods (Vukmirović, Pupavac, 2013).

Object program for the algorithm of Exhaustive search algorithm in the spreadsheet interface explores and finds all relations with the minimal value achieved. Also, the program can explore and find relations with values close to optimal (minimal) value with predefined minimal deviation. Crucial factor for structuring a transport network with transportation at minimal cost, maximal profits and minimal time is the use of relevant information technologies and computer applications that allow the calculation of the optimal connectivity of nodes (towns) and scheduling of transport relations. Methodological frame of use of Visual Basic as a development tool in the visual modelling of Exhaustive Search Algorithm in VBA for Excel can serve as an incentive in creating new highly sophisticated algorithms, which will enable us to compute optimal and suboptimal solutions of transport network (Vukmirović and Pupavac, 2013), (Vukmirović, Čičin-Šain and Host, 2015).

The transport network model based on a quadratic pyramid is used to prove and analyze the possibilities of achieving a large number of optimal solutions with the same or similar values (Vukmirović, Pupavac, 2013.; Vukmirović, Čičin-Šain, Host, 2015; Vukmirović, Čapko, Babić, 2019a). Table 1 shows the application of the Exhaustive Search algorithm on the example of the base of a square pyramid, using the Visual Basic for Excel program, the spreadsheet interface.

In the address range N1:V9, the distance values between the nodes of the square pyramid are entered. In the address range A1:J20160, all possible solutions for the transport network in the form of a square pyramid, consisting of 9 nodes, were calculated. Since the matrix shown in the address range N1:V9 is symmetric, 20160 possible solutions were calculated, sorted by the value of the transport path from the smallest to the largest value. The smallest value of the transport path represents the

optimal solution. The Table 1 shows that 18 optimal solutions were calculated and found, where the value is 41.

Table 1 Exhaustive search algorithm in spreadsheet interface

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1	1	2	5	6	8	9	7	3	4	1	41	0,0%		1,0	5,0	5,0	3,5	7,0	10,5	11,0	11,0	14,0	1	A
2	1	2	5	8	6	9	7	3	4	1	41	0,0%		5,0	1,0	7,0	3,5	5,0	8,0	10,0	7,0	11,0	2	B
3	1	2	5	8	9	6	7	3	4	1	41	0,0%		5,0	7,0	1,0	3,5	5,0	8,0	7,0	10,0	11,0	3	C
4	1	2	8	9	7	6	5	3	4	1	41	0,0%		3,5	3,5	3,5	1,0	3,5	7,0	8,0	8,0	10,5	4	D
5	1	2	8	9	6	7	5	3	4	1	41	0,0%		7,0	5,0	5,0	3,5	1,0	3,5	5,0	5,0	7,0	5	E
6	1	2	8	6	9	7	5	3	4	1	41	0,0%		10,5	8,0	8,0	7,0	3,5	1,0	3,5	3,5	3,5	6	F
7	1	2	8	6	9	7	5	4	3	1	41	0,0%		11,0	10,0	7,0	8,0	5,0	3,5	1,0	7,0	5,0	7	G
8	1	2	8	9	6	7	5	4	3	1	41	0,0%		11,0	7,0	10,0	8,0	5,0	3,5	7,0	1,0	5,0	8	H
9	1	2	8	9	7	6	5	4	3	1	41	0,0%		14,0	11,0	11,0	10,5	7,0	3,5	5,0	5,0	1,0	9	I
10	1	2	4	5	6	8	9	7	3	1	41	0,0%		1	2	3	4	5	6	7	8	9		
11	1	2	4	5	8	9	6	7	3	1	41	0,0%		A	B	C	D	E	F	G	H	I		
12	1	2	4	5	8	6	9	7	3	1	41	0,0%												
13	1	3	7	9	6	8	5	2	4	1	41	0,0%												
14	1	3	7	6	9	8	5	2	4	1	41	0,0%												
15	1	3	7	9	8	6	5	2	4	1	41	0,0%												
16	1	3	5	7	6	9	8	2	4	1	41	0,0%												
17	1	3	5	7	9	6	8	2	4	1	41	0,0%												
18	1	3	5	6	7	9	8	2	4	1	41	0,0%												
19	1	2	8	6	9	7	3	5	4	1	43	4,9%												
20	1	2	8	9	6	7	3	5	4	1	43	4,9%												
21	1	2	3	7	6	9	8	5	4	1	43	4,9%												
22	1	2	3	7	9	6	8	5	4	1	43	4,9%												
49	1	3	7	6	9	8	2	5	4	1	43	4,9%												
50	1	3	7	9	6	8	2	5	4	1	43	4,9%												
51	1	2	5	8	9	7	6	3	4	1	43,5	6,1%												
52	1	2	5	7	9	8	6	3	4	1	43,5	6,1%												
20157	1	6	2	7	5	3	8	4	9	1	81	97,6%												
20158	1	9	4	7	2	5	8	3	6	1	81	97,6%												
20159	1	9	4	8	3	5	7	2	6	1	81	97,6%												
20160	1	6	3	8	5	2	7	4	9	1	81	97,6%												

Source: Authors

In column L, in the address area L1:L20160, deviations from the optimal value are calculated. In the example, a value of 5% of the permissible deviation is defined. The Table 1 shows that 32 suboptimal solutions were calculated within the address area A19:J50, the value of which is within the permissible deviation of 5%.

3. URBAN AGLOMMERATION RIJEKA AND TRANSPORTATION NETWORK

The Development Strategy of the Urban Agglomeration Rijeka is based on strategic and territorial documents of all cities and municipalities of the Urban Agglomeration Rijeka and is linked to all higher order strategic documents (Grad Rijeka, 2017). According to the Guidelines for the Establishment of Urban Areas and the Urban Development Strategy, the City of Rijeka has proposed a change in the scope of the Rijeka agglomeration for the period 2021-2027 and submitted to the

Ministry the final proposal for the scope of the Urban Agglomeration of Rijeka according to which the Rijeka agglomeration would include 14 units of local self-government: The cities of Rijeka, Bakar, Kastav, Kraljevica and Opatija, and the municipalities of Čavle, Jelenje, Klana, Kostrena, Lovran, Matulji, Mošćenička Draga, Omišalj and Viškovo. The proposal also included the approval of the accession of the above-mentioned cities and municipalities to the system of urban agglomeration (Grad Rijeka, 2021).

The relations between the port and the city are thus variable, both spatially and over time, and the idea of the interdependence of port activities and urban phenomena has been re-emerging throughout the history and around the world. By studying the concept of the port-city interface, it is concluded that the relationship between the port and the city, despite the fact that they are separated, becomes increasingly intertwined and complex as new changes affecting both the port and the city are constantly emerging (Jugović, Sirotić, Peronja, 2021).

Within the port and city interface, however, there are specialized business activities that are related to the port, shipping, and the city through various types of transactions (e.g. finance, risk management, consulting, etc.) (Zhao et al., 2017 as cited in Jugović, Sirotić, Peronja, 2021).

The port-city interface can be described as a system, as a concept, or as a system of mechanisms that, together and individually, connect the port and the city.» (Hoyle, 2006). This intermediate zone is a 'threshold' (Crotti, 2000 as cited in Brambilla, Laine, Bocchi, 2015) with variable thickness and configurations that vary depending on a number of factors. The research of this system offers opportunities for setting strategies for urban-port territories and in this context for the development of urban agglomeration.

Global positioning of Rijeka with the development of the Rijeka Traffic Direction as the Strategic Objective 1 directly contributes to the strengthening of Rijeka's competitiveness on a global level, with its port as the greatest comparative advantage. The development of the Port of Rijeka involves a whole range of supporting services with a trend of enhancing the port logistics chains. This implies linking and aligning all entities within the transport sector. The purpose of such linking is to develop the Rijeka Traffic Direction as the unique economic offer on the global market (Strategija razvoja grada Rijeke, 2013.)

Based on the conceptual model of the transport network of the Urban Agglomeration of Rijeka, a flexible and adaptive transport network has been designed, for which it can be assumed that in addition to the optimal solution, there may be several suboptimal solutions. The city of Rijeka is defined as the origin and destination of the flexible transport network (start/end city). By identifying and scientifically based analysis of the problem of more optimal and suboptimal relations, a significant impact on the synergy of the parameters of success and efficiency of the transport network of the Rijeka Urban Agglomeration can be achieved, and thus on a better connection of the Port of Rijeka and the city of Rijeka with the cities and municipalities of the Rijeka Urban Agglomeration.

4. CONCEPTUAL MODEL OF TRANSPORTATION NETWORK OPTIMIZATION ON THE EXAMPLE OF THE URBAN AGGLOMERATION OF RIJEKA

Urban Agglomeration of Rijeka consists of 14 cities and municipalities, which represent 14 transport nodes or junctions in the transport network as explained in Chapter 3. Table 2 shows the distances between the cities and municipalities of the Rijeka agglomeration. Google Maps technology was used as a data source for the distances. Google Maps is a digital mapping platform that offers various services such as satellite imagery, route planning, location solutions, etc. and provides an automatic graphical representation of the selected route on a digital map.

Table 2 Distances between cities and municipalities of Urban Agglomeration of Rijeka

No	City	Abb	Distances													
1	Rijeka	RI	0	22	12	7	14	12	9	11	9	18	27	21	28	15
2	Kraljevica	KR	22	0	8	15	37	33	31	32	17	40	6	49	57	31
3	Bakar	BA	12	8	0	7	27	23	20	22	9	29	13	39	47	15
4	Kostrena	KO	7	15	7	0	22	26	23	24	12	32	20	35	43	18
5	Opatija	OP	14	37	27	22	0	4	12	7	22	20	41	7	14	28
6	Matulji	MA	12	33	23	26	4	0	8	3	19	15	38	10	19	25
7	Viškovo	VI	9	31	20	23	12	8	0	5	16	10	34	19	26	11
8	Kastav	KA	11	32	22	24	7	3	5	0	18	13	36	14	22	14
9	Čavle	ČA	9	17	9	12	22	19	16	18	0	22	22	30	37	6
10	Klana	KL	18	40	29	32	20	15	10	13	22	0	44	29	36	16
11	Omišalj	OM	27	6	13	20	41	38	34	36	22	44	0	47	55	35
12	Lovran	LO	21	49	39	35	7	10	19	14	30	29	47	0	10	43
13	Mošćenička Draga	MO	28	57	47	43	14	19	26	22	37	36	55	10	0	50
14	Jelenje	JE	15	31	15	18	28	25	11	14	6	16	35	43	50	0
			RI	KR	BA	KO	OP	MA	VI	KA	ČA	KL	OM	LO	MO	JE

Source: Authors

The conceptual model of optimization of the transport network of Urban agglomeration Rijeka, was designed in the function of finding suboptimal solutions. **The solution of the transport network was calculated using the programming language for mathematical modeling FICO Xpress – solver for linear and quadratic programming with continuous or integer variables (Figure 1).**

Figure 1 Solution of transport network has been calculated by usage of programming language for mathematical modeling Xpress

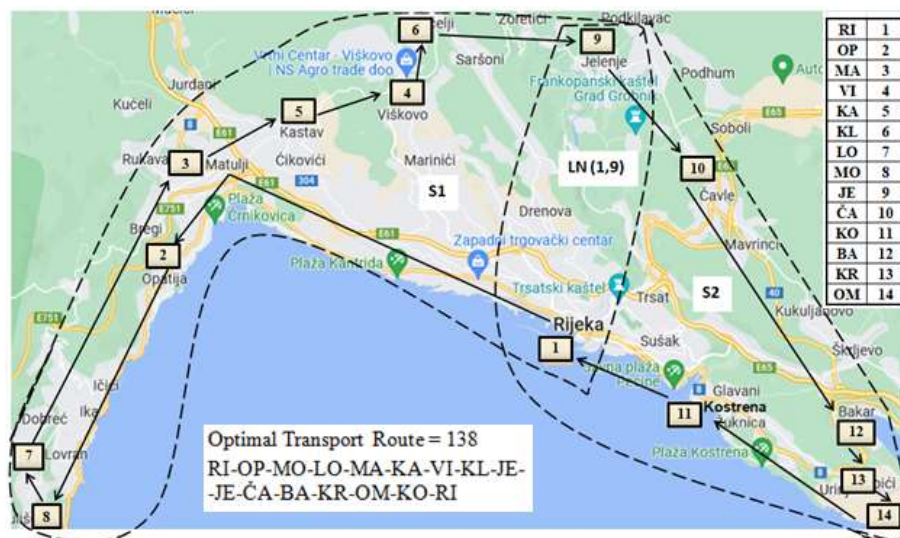


Source: Authors

On the Map 1 is a graphical representation of the optimal transport route and the conceptual model of the transport network of the Urban Agglomeration of Rijeka. In the Table 2 in Map 1, the right part shows the names of the cities and the abbreviations of the cities and their numbers, and the lower part shows the value of the optimal solutions and configurations (direction) of the optimal transport route.

In accordance with the calculated optimal solution presented in Map 1, a conceptual model of the transport network of the Rijeka agglomeration was designed in order to find possible optimal and suboptimal solutions. It is considered that in the exhaustive search algorithm used in this work, the maximum number of nodes is 9.

Map 1 Graphical display of the optimal transport route and the conceptual model of the transportation network of Urban agglomeration Rijeka



Source: Authors

The conceptual model of the transport network of is divided into two groups (segments) S1 and S2 from the point of view of finding optimal and suboptimal solutions (Vukmirović, Čapko, Babić, 2019a). The model shows that segment S1 includes nodes (cities and municipalities) from 1 to 9 and segment S2 includes nodes 9 to 14 and node 1. In segments S1 and S2, the possibilities of connecting cities (nodes) via different transport routes were analyzed and optimal and suboptimal solutions were calculated using an exhaustive search algorithm. The example also defines nodes 1 and 9 (cities of Rijeka and Jelenje), which are common to the defined segments of the S1 and S2 transport network. Nodes 1 and 9, marked in Map 1 as LN (1,9), represent in the model the common points and at the same time the end points for the S1 and S2 transport network segments and enable the connection of the transport network segments.

Table 2a shows data of estimated transport times between cities and municipalities (nodes) of the Urban Agglomeration of Rijeka, which were collected in a certain time interval, using the Google Maps tool.

Table 2a Estimated Times between cities and municipalities of Rijeka Urban Agglomeration

No	City	Abb	Estimated Times													
			1	24	19	16	27	22	21	22	17	31	26	40	52	24
1	Rijeka	RI	1	24	19	16	27	22	21	22	17	31	26	40	52	24
2	Kraljevica	KR	24	1	12	16	34	28	28	30	21	36	13	48	59	28
3	Bakar	BA	19	12	1	13	31	26	26	27	17	33	20	46	58	25
4	Kostrena	KO	16	16	13	1	30	24	24	26	16	32	22	45	57	23
5	Opatija	OP	27	34	31	30	1	9	23	15	25	28	37	19	31	32
6	Matulji	MA	22	28	26	24	9	1	18	9	21	21	33	29	41	28
7	Viškovo	VI	21	28	26	24	23	18	1	12	19	14	31	37	48	20
8	Kastav	KA	22	30	27	26	15	9	12	1	24	15	35	33	46	24
9	Čavle	ČA	17	21	17	16	25	21	19	24	1	26	24	39	51	7
10	Klana	KL	31	36	33	32	28	21	14	15	27	1	39	46	56	28
11	Omišalj	OM	26	13	20	22	37	33	31	35	24	39	1	51	62	30
12	Lovran	LO	40	48	46	45	19	29	37	33	38	46	51	1	16	45
13	Mošćenička	MO	52	59	58	57	31	41	48	46	47	56	62	16	1	54
14	Jelenje	JE	24	28	25	23	32	28	20	24	8	28	30	45	54	1
			RI	KR	BA	KO	OP	MA	VI	KA	ČA	KL	OM	LO	MO	JE

Source: Authors

5. EXHAUSTIVE SEARCH ALGORITHM APPROACH TO FINDING THE OPTIMAL AND SUBOPTIMAL SOLUTIONS OF THE TRANSPORTATION NETWORK

An Exhaustive Search algorithm developed in Visual Basic was used in the optimization of the transport network of Urban Agglomeration of Rijeka. Object-

oriented programming in Visual Basic was used to create and visualize the Exhaustive Search Algorithm in order to calculate one or more optimal transport routes. The cities included in the transportation network calculation belong to the S1 and S2 node groups explained in the previous chapter and shown in Map 2. Table 4 and Table 5 contain the results for the given example.

Table 4 shows the optimal solution for the first segment (S1) of the transport network using the exhaustive search algorithm. The results were calculated using the Visual Basic in Excel (VBA for Excel) program created by the authors of this paper. It can be seen from Table 3 that 20160 possible transport routes (relations) were calculated, where the minimum route length is 82 km.

Table 5 shows the optimal solution of the second segment (S2) of the transport network according to the method of exhaustive search using the Visual Basic program in the Excel spreadsheet interface. It can be seen from Table 4 that four optimal transport routes (relations) with a minimum route length of 56 km were calculated (rows 1-4).

The exhaustive search algorithm used in this work resulted in a solution with a maximum of 9 nodes. The Table 5 shows cities and municipalities as nodes representing transport nodes and ordinal numbers of nodes from 1 to 9. Nodes 1 and 9 (cities of Rijeka and Jelenje) are common to both city segments (transport network segments) defined and shown on Map 2, representing the end points of the city segment.

Table 4 Solution of the first segment (S1) of the transport network

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	
1	Nodes								Length	Time	Distances								City	No.
2	1	9	10	14	13	12	11	1	56	85	0	22	12	7	9	27	0	RI	1	
3	1	9	10	13	14	12	11	1	56	90	22	0	8	15	17	6	31	KR	13	
4	1	9	10	12	13	14	11	1	56	87	12	8	0	7	9	13	15	BA	12	
5	1	9	10	12	14	13	11	1	56	89	7	15	7	0	12	20	18	KO	11	
6	1	9	10	11	14	13	12	1	64	89	9	17	9	12	0	22	6	ČA	10	
7	1	9	10	11	13	14	12	1	64	91	27	6	13	20	22	0	35	OM	14	
8	1	9	10	11	12	13	14	1	66	87	0	31	15	18	6	35	0	JE	9	
9	1	9	10	11	12	14	13	1	66	93	RI	KR	BA	KO	ČA	OM	JE			
10	1	9	10	13	14	11	12	1	68	95	1	13	12	11	10	14	9			
11	1	9	10	14	13	11	12	1	68	92										
12	1	9	11	13	14	12	10	1	70	106	OPTIMAL TRANSPORT ROUTE								Length	
13	1	9	11	14	13	12	10	1	70	104	1	9	10	14	13	12	11	1	56	
14	1	9	11	12	14	13	10	1	70	107	RI	JE	ČA	OM	KR	BA	KO	RI		
15	1	9	11	12	13	14	10	1	70	102		0	+6	+22	+6	+8	+7	+7	=56	
16	1	9	10	12	11	14	13	1	70	96										
357	1	14	10	12	11	9	13	1	136	155	OPTIMAL TRANSPORT ROUTE								Time	
358	1	12	11	10	14	9	13	1	141	154	1	9	10	14	13	12	11	1	85	
359	1	12	11	10	13	9	14	1	141	153	RI	JE	ČA	OM	KR	BA	KO	RI		
360	1	12	10	11	13	9	14	1	141	152		0	+7	+24	+13	+12	+13	+16	=85	
361	1	12	10	11	14	9	13	1	141	156										

Source: Authors

Table 5. Solution of the second segment (S2) of the transport network

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	
1	Nodes										Length	Time	Distances										No.	City
2	1	2	8	7	3	5	4	6	9	1	82	166	0	14	12	9	11	18	21	28	0	1	RI	
3	1	7	8	2	3	5	4	6	9	1	83	159	14	0	4	12	7	20	7	14	28	2	OP	
4	1	8	7	2	3	5	4	6	9	1	83	159	12	4	0	8	3	15	10	18	25	3	MA	
5	1	2	7	8	3	5	4	6	9	1	83	166	9	12	8	0	5	10	19	26	11	4	VI	
6	1	3	7	8	2	5	4	6	9	1	84	167	11	7	3	5	0	13	14	22	14	5	KA	
7	1	2	8	7	3	5	6	4	9	1	85	161	18	20	15	10	13	0	29	36	16	6	KL	
8	1	3	2	8	7	5	4	6	9	1	85	165	21	7	10	19	14	29	0	10	43	7	LO	
9	1	3	8	7	2	5	4	6	9	1	85	167	28	14	18	26	22	36	10	0	50	8	MO	
10	1	2	7	8	3	5	6	4	9	1	86	161	0	28	25	11	14	16	43	50	0	9	JE	
11	1	8	7	2	3	5	6	4	9	1	86	160	RI	OP	MA	VI	KA	KL	LO	MO	JE			
12	1	7	8	2	3	5	6	4	9	1	86	160	1	2	3	4	5	6	7	8	9			
13	1	5	2	8	7	3	4	6	9	1	86	173												
14	1	4	5	2	8	7	3	6	9	1	86	173	OPTIMAL TRANSPORT ROUTE										Length	
15	1	5	3	7	8	2	4	6	9	1	86	172	1	2	8	7	3	5	4	6	9	1	82	
16	1	3	2	7	8	5	4	6	9	1	86	166	RI	OP	MO	LO	MA	KA	VI	KL	JE	RI		
17	1	4	5	3	7	8	2	6	9	1	87	174		14	+14	+10	+10	+3	+5	+10	+16	+0	82	
40317	1	3	6	8	9	7	4	2	5	1	205	273												
40318	1	5	7	9	8	6	2	4	3	1	206	279	SUBOPTIMAL TRANSPORT ROUTE										Length	
40319	1	3	4	8	9	7	6	2	5	1	206	276	1	7	8	2	3	5	4	6	9	1	83	
40320	1	3	4	7	9	8	6	2	5	1	206	275	RI	OP	MA	VI	KA	KL	LO	MO	JE	RI		
40321	1	5	8	9	7	6	2	4	3	1	207	282												

Source: Authors

Table 6 presents the consolidated optimal and suboptimal solutions of the transport network in which segments S1 and S2 of the transport network are connected and that were calculated values of minimum distances (column Length) and estimated transport times (column Time). In the segment Optimal and suboptimal transportation routes, the abbreviations denote the cities and municipalities of the Urban agglomeration of Rijeka, as defined in table 2. The city of Rijeka (RI) is defined and presented as a start/end node.

Table 6 Consolidated solutions of the transport network

No.	Optimal and suboptimal transport routes															Length	Time
1	RI	OP	MO	LO	MA	KA	VI	KL	JE	ČA	BA	KR	OM	KO	RI	138	253
2	RI	OP	MO	LO	MA	KA	VI	KL	JE	ČA	KR	OM	BA	KO	RI	138	256
3	RI	OP	MO	LO	MA	KA	VI	KL	JE	ČA	OM	KR	BA	KO	RI	138	251
4	RI	OP	MO	LO	MA	KA	VI	KL	JE	ČA	BA	OM	KR	KO	RI	138	255
5	RI	LO	MO	OP	MA	KA	VI	KL	JE	ČA	OM	KR	BA	KO	RI	139	244
6	RI	LO	MO	OP	MA	KA	VI	KL	JE	ČA	KR	OM	BA	KO	RI	139	249
7	RI	LO	MO	OP	MA	KA	VI	KL	JE	ČA	BA	KR	OM	KO	RI	139	246
8	RI	LO	MO	OP	MA	KA	VI	KL	JE	ČA	BA	OM	KR	KO	RI	139	248
9	RI	MO	LO	OP	MA	KA	VI	KL	JE	ČA	OM	KR	BA	KO	RI	139	244
10	RI	MO	LO	OP	MA	KA	VI	KL	JE	ČA	KR	OM	BA	KO	RI	139	249
11	RI	MO	LO	OP	MA	KA	VI	KL	JE	ČA	BA	KR	OM	KO	RI	139	246
12	RI	MO	LO	OP	MA	KA	VI	KL	JE	ČA	BA	OM	KR	KO	RI	139	248
13	RI	OP	LO	MO	MA	KA	VI	KL	JE	ČA	OM	KR	BA	KO	RI	139	251
14	RI	OP	LO	MO	MA	KA	VI	KL	JE	ČA	KR	OM	BA	KO	RI	139	256
15	RI	OP	LO	MO	MA	KA	VI	KL	JE	ČA	BA	KR	OM	KO	RI	139	253
16	RI	OP	LO	MO	MA	KA	VI	KL	JE	ČA	BA	OM	KR	KO	RI	139	255
17	RI	MA	LO	MO	OP	KA	VI	KL	JE	ČA	OM	KR	BA	KO	RI	140	252
18	RI	MA	LO	MO	OP	KA	VI	KL	JE	ČA	KR	OM	BA	KO	RI	140	257
19	RI	MA	LO	MO	OP	KA	VI	KL	JE	ČA	BA	KR	OM	KO	RI	140	254
20	RI	MA	LO	MO	OP	KA	VI	KL	JE	ČA	BA	OM	KR	KO	RI	140	256
21	RI	OP	MO	LO	MA	KA	KL	VI	JE	ČA	OM	KR	BA	KO	RI	141	246
22	RI	OP	MO	LO	MA	KA	KL	VI	JE	ČA	KR	OM	BA	KO	RI	141	251
23	RI	OP	MO	LO	MA	KA	KL	VI	JE	ČA	BA	KR	OM	KO	RI	141	248
24	RI	OP	MO	LO	MA	KA	KL	VI	JE	ČA	BA	OM	KR	KO	RI	141	250

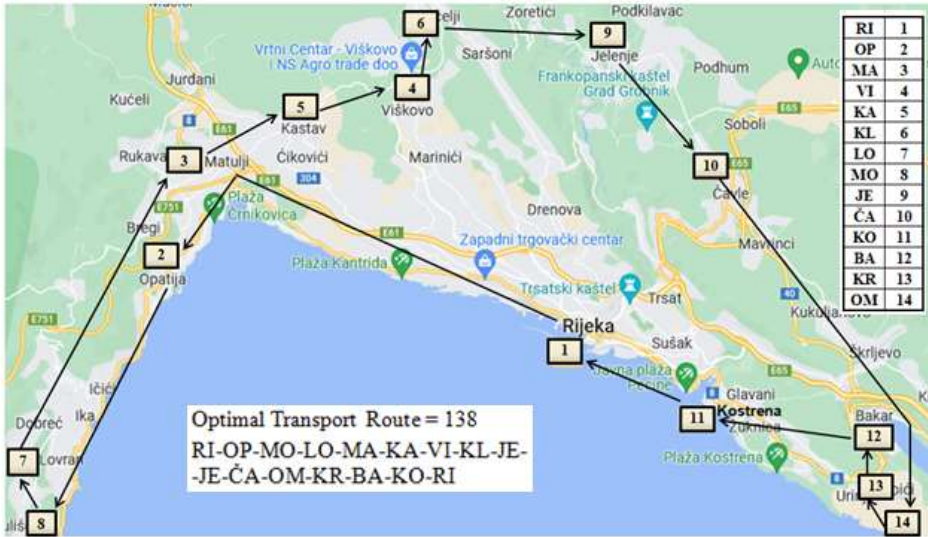
Source: Authors

In Table 6, it can be seen that four optimal relations and twenty suboptimal relations were calculated. In column No. ordinal numbers from 1 to 4 indicate optimal relations, and ordinal numbers from 5 to 24 indicate suboptimal relations. By comparing the optimal solution, which represents the minimum length and configuration of the transport route, calculated by the exhaustive search algorithm in VBA for Excel (Table 4, column No., row 1), with the optimal solution obtained using the mathematical modeling program Xpress (Figure 1), it can be observed that the results match. From the point of view of the minimum distance criterion, 4 optimal solutions were calculated.

From the point of view of the minimum distance criterion, 4 optimal solutions were calculated. Suboptimal solutions were calculated and selected from the viewpoint of minimum distance and minimum transport time criteria. From the point of view of the minimum distance, the deviation values range within 3 km, that is, 2.5% of the deviation interval from the calculated total minimum length of the route. From the point of view of minimum time, the values of suboptimal relations are less than or equal to the time values of optimal relations.

Map 2 shows the optimal solution of the transport network of Urban Agglomeration of Rijeka, which corresponds to the solution calculated in Table 5 and shown in row 1 (No.1). Map 3 shows an example of another optimal solution of the transport network, which corresponds to the solution calculated in Table 5 and shown in line 3 (No.3).

Map 2 Example of another optimal transport route



Source: Authors

Map 3 presents the suboptimal solution of the transport network that corresponds to the solution calculated in Table 5 and shown in line 9 (No. 9).

Map 3 Suboptimal transport route



Source: Authors

Comparing the optimal solutions on Map 2 and the suboptimal solution on Map 3, we can see the change in the order of cities (nodes) in segments S1 and S2 of the transport network of the Rijeka. By considering several optimal and suboptimal solutions within the interval of a given deviation, it is possible to identify and analyze the synergy of all relevant factors that determine the best (optimal) solution or a set of best solutions (optimal and suboptimal solutions). The change can be observed in the route depicted on Map 2, which goes from Rijeka to Opatija, passing over Opatija to Mošćenička Drag and Lovran. On Map 3, it can be seen that the route goes from Rijeka to Mošćenička Draga and Lovran, then to Opatija, and to Matulji. By identifying more optimal and suboptimal routes, it is possible to achieve a significant impact on the synergy of the parameters of transport performance and efficiency from the origin to the destination: minimum travel time, minimum time, minimum cost and maximum utilization of transport capacity.

6. SIGNIFICANCE OF CALCULATING AND FINDING MORE OPTIMAL AND SUBOPTIMAL SOLUTIONS OF THE TRANSPORT NETWORK TO THE BUSINESS EFFECTS

When considering the meaning of suboptimal solutions, it is necessary to distinguish between two problem situations: 1) a situation in which the optimal solution is the most favorable and 2) a situation in which a suboptimal solution can achieve better effects compared to the optimal solution. It should be noted that it is always possible to calculate a suboptimal solution. The Traveling Salesman Problem

(TSP) is a problem which requires an optimal solution, especially if the route is to be used several times. In general, if the solution is to be applied only once, a suboptimal solution will be adequate and a very close to optimal solution may be even more desirable than the optimal solution (Gregory, 1970). Sequential insertion with possible requests for variable quotes to all trucks and to all routes potentially produces suboptimal solutions (Greenwood, 2009).




In Table 6, it can be seen that four optimal relations and twenty suboptimal relations were calculated. In column No. ordinal numbers from 1 to 4 indicate optimal relations, and ordinal numbers from 5 to 24 indicate suboptimal relations. From the point of view of the minimum distance criterion, 4 optimal solutions were calculated. Suboptimal solutions were calculated and selected from the viewpoint of minimum distance and minimum transport time criteria. From the point of view of the minimum distance, the deviation values range within 3 km, that is, 2.5% of the deviation interval from the calculated total minimum length of the route. From the point of view of minimum time, the values of suboptimal relations are less than or equal to the time values of optimal relations.

It can be seen from the Table 6. that the relationships between the calculated distance values and the time values are not proportional. Also, it should be noted that distance values are fixed, and time values are variable, that is, subject to change. This means that in different periods of time significant changes in the duration of transport are possible for transport routes. **In the transportation network model in Table 6, four optimal solutions were calculated from the point of view of minimum distance. Considering the criteria of distance and time, the optimal solution in Table 6 is transportation route No. 1. However, unlike the distance values, which are fixed, the transportation times change, so in a different time period, the minimum time for a different transportation route can be calculated.**

Transport time is an important dimension in the evaluation of transport costs, particularly since logistics concomitantly involves cost and time management. Transport time is an important dimension in evaluating transport costs, particularly since logistics concomitantly involves cost and time management. The major time-related elements are (Rodrigue, 2013):

- 📁 **Transport time.** Concerns the real duration of transport, which tends to be easily understood since commonly a proportional function of distance. Geographical constraints such as weather or technical limitations such as operational speed have a direct impact on transport time. Transport time on roads is technically limited to legal speed limits. For maritime and air, the limitation mainly concerns fuel economy and design speed. Although rail can accommodate a variety of speeds, tight schedules impose limited variations in operational speeds.
- 📁 **Order time.** Almost all transport requires a form of advance preparation, mainly to secure a capacity, an itinerary and a rate. In some cases, the order time is short and a matter of queuing on a basis, while in other cases orders have to be secured months in advance.
- 📁 **Timing.** Involves the usage of a specific departure time, which depending on the mode can have a level of flexibility. While for air and rail travel timing is commonly tight due to fixed schedules and access to a terminal capacity (such as a gate and a take off time), commuters and trucking have more flexibility. If there

is congestion either at the origin, destination or in between, trucking companies may elect to modify their schedule accordingly (earlier or later delivery).

-  **Punctuality.** Represents the ability to keep a specified schedule, which can be represented as an average deviation from a scheduled arrival time. The longer the distance, the more likely are potential disruptions that may affect schedule integrity. Some movements may have a level of tolerance to disruptions in punctuality while others, such as heading to a business meeting or flows in a just-in-time supply chain, have limited tolerance.
-  **Frequency.** The number of departures for a specific time range. The higher the frequency, the better the level of service. However, a high frequency ties up a larger quantity of vehicles. Distance is also a factor for lower frequency since transport demand tends to decline accordingly. Combining long distance travel and high frequency is an expensive undertaking for transport providers as a greater number of vehicles must be assigned to a specific route, as in the case of maritime container shipping.
-  **Optimal speed for fuel economy.** According to the United States Department of Energy, fuel economy is the highest at driving between 35 and 60 mph. At higher or lowest speed, the fuel efficiency drops, costing an additional value per litre (Davis, Boundy, 2012), (Ahmad, 2022). That means that in the case of choosing a longer (suboptimal) transport route on which the estimated driving time is shorter, lower or equal fuel consumption is possible compared to a shorter (optimal) transport. By comparing the optimal and suboptimal solution in Table 6, No.2 (optimal) and No.6 (suboptimal), we may see that time for suboptimal solution is 12 minutes shorter than time for optimal solution.

In the example of the transport network optimization of Urban agglomeration Rijeka, the criterion (factor) of optimization is the minimum length of the transport relation. Considering the more optimal solutions within a given deviation interval, it is possible to parse and analyze the synergy of all relevant factors that determine the best (optimal) or set of best solutions (transport cost and time, choosing the alternative route in different cases of traffic jams, punctuality, frequency and possibilities for the greater utilization of transport capacity).

7. CONCLUSION

It was proved the hypothesis that the visual and object-oriented methods of modelling and programming in the spreadsheet interface, on the base of Exhaustive Search Algorithm for solving Traveling Salesman Problem, enable transport network optimization that identify multiple optimal and suboptimal solutions and have significant influence on the business effects of transport in Rijeka Urban Agglomeration: reduction of transport time and costs, greater safety of transport within the time schedules and better utilization of vehicle capacity. Based on the conceptual model of the transport network of the Urban Agglomeration of Rijeka, it is possible to define a transport network segment for which it can be assumed that there can be several solutions of approximately equal value in an optimal solution. Based on the conceptual model, a route redundancy problem can also be defined,

which can generate further optimal (minimum) values for the transport route. By identifying and performing a scientifically sound analysis of the problem of multiple optimal and suboptimal routes, a significant impact on the synergy of transportation performance and efficiency parameters from origin to destination can be achieved: minimum distance traveled, shortest time at minimum cost, and maximum utilization of transportation capacity.

The implementation of an Exhaustive Search algorithm in Visual Basic for optimizing the transport network of the Urban Agglomeration of Rijeka has demonstrated its potential to address numerous transportation challenges. By utilizing object-oriented programming and visualization techniques, this algorithm efficiently calculated one or more optimal transport routes within the network. The results obtained from this approach, as presented in Table 4 and Table 5, showcased the effectiveness of the algorithm in finding optimal solutions with minimal route lengths. The exhaustive search algorithm's ability to calculate a vast number of possible transport routes, as evidenced by the 20,160 calculated relations in Table 4, demonstrates its comprehensive exploration of the network.

Additionally, the identification of multiple optimal routes with varying lengths in Table 5 underscores the algorithm's flexibility in accommodating diverse transportation needs.

By utilizing this approach, transportation planners and businesses can benefit from the algorithm's ability to optimize route planning, resulting in more efficient and cost-effective transportation operations. In this research the algorithm's successful implementation, considering up to 9 nodes, indicates its scalability for handling larger transport networks. Overall, the utilization of the Exhaustive Search algorithm in this study has shown promising results in addressing transportation challenges, enhancing route optimization, and positively impacting various aspects businesses and urban development, including possibility for cost reduction, time efficiency, resource utilization, environmental sustainability, improved urban planning, infrastructure investment, economic growth, and enhanced urban livability.

In this paper, research on the optimization of the transportation network is limited to the parameters of distance and time. The objective of the research on the optimization of the transportation network based on the analysis of transportation distance and time was to consider the importance of calculating suboptimal solutions in order to find alternative transportation routes. In future research, the transportation network optimization model can be extended to other optimization parameters such as transportation costs, transportation capacity optimization, and variable transportation conditions.

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