THE INFLUENCE OF CONFIDENCE LEVEL, CORRELATION AND VOLATILITY ON VALUE AT RISK. SIX CASE STUDIES

Izabela Pruchnicka-Grabias, Ph.D.¹
¹Warsaw School of Economics, Republic of Poland, ipruch@wp.pl

Abstract

Studies show that hedge funds and other financial institutions often apply the standard deviation as a risk measure. Even if one looks at hedge fund internet pages with investment results data for investors, they usually present them with standard deviations and Sharpe indicators, neglecting the fact that their investment assets are not always normally distributed, as well as such important measures as for example kurtosis and skewness. The author estimates the correlation and volatility for selected investment assets and verifies assumptions of popular risk models concerning these parameters. The impact of the confidence level, correlation and volatility on Value at Risk is analyzed.

JEL Classification: E44, O16

Keywords: hedge funds, financial institutions, standard deviation, investment assets

INTRODUCTION

Studies show that hedge funds and other financial institutions often apply the standard deviation as a risk measure. Even if one looks at hedge fund internet pages with investment results data for investors, they usually present them with standard deviations and Sharpe indicators, neglecting the fact that their investment assets are not always normally distributed, as well as such important measures as for example kurtosis and skewness defined by the following formulas.

\[
\text{Kurtosis} = \frac{T(T + 1)}{(T - 1)(T - 2)(T - 3)} \sum_{t=1}^{T} \left( \frac{R_{t} - \bar{R}}{\sigma} \right)^{4} - \frac{3(T - 1)^{2}}{(T - 2)(T - 3)},
\]

where:
T – the number of observations

σ – the standard deviation of rates of return

$\bar{R}$ – arithmetic mean of rates of return

Skewness is the third central moment of a distribution and measures the symmetry of a return distribution around the mean. Mathematically it is calculated as:

$$\text{Skewness} = \frac{T}{(T-1)(T-2)} \sum_{t=1}^{T} \left( \frac{R_{t} - \bar{R}}{\sigma} \right)^3$$

Taking the assumption that the volatility does not change in time leads to inadequate results. It has been well documented in the literature that it changes. Thus, it also influences value at risk. The key matter for calculating VaR is the choice of the holding period. Holding period is understood as the period in which the calculated loss may be generated. The proper choice of the holding period must be based on the time in which an institution is able to sell the majority of liquid assets. Banks usually use a one day holding period, however for hedge funds it is not adequate, because their assets are less liquid. For this reason, they often use a 10-day holding period.

Another weak point of this method is using the square root of time. Although it is widely accepted in practice, F. Duc and Y. Schorderet show that the approximation of VaR using the square root of time rule differs significantly from the correct VaR and makes it impossible to catch the fact that the risk starts to fall down from the sixth month.

Value at risk in the given confidence interval can be written as:

$$\text{VaR} = V \times P \times S$$

where:

1 F.S. Lhabitant, Handbook of hedge funds, John Wiley & Sons, Ltd., Chichester 2006, p. 437. Note that some analysts do not subtract the second term from the kurtosis. As a result, when T is large, the threshold value for the normal distribution becomes 3 rather than 0.


4 P. Best, Wartość narażona na ryzyko. Obliczanie i wdrażanie modelu VaR [Value at risk. Calculating and implementing the VaR model], Dom Wydawniczy ABC, Kraków 2000, p. 27.
V – variance of rates of returns of the asset
P – the value of the investment
S – the number of standard deviations below the average

Case study I

Let’s assume that a hedge fund invests its assets of 100000000 USD in crude oil futures. Let’s calculate value at risk for different confidence levels for a 10-day holding period, provided that the daily volatility is counted for the period 2005–2010 (see chart 1).

Chart 1. The impact of the confidence level on VaR changes for crude oil daily rates of return.

Source: Author’s calculations.

The positive relation between the confidence level and value at risk is not a surprise, but it is worth emphasizing that the higher the confidence level is, the higher is the sensitivity of VaR changes to it. Thus, these are especially high levels of confidence at which the risk management is the most difficult and mistakes are most severe. Besides, the standard deviation level taken for the above calculations was counted for the six-year period starting from 2005 and ending with 2010. If it was calculated for a three, four or five-year period, the result would probably change.
Another measure that influences the VaR value is volatility. It is shown beneath that one-day volatility levels for different periods of time differ from each other. As it is depicted in chart 2, one-day volatility for six-year data is 2.46. It reaches its peak for three-year data and falls down up to one-year data.

**Chart 2.** Crude oil futures volatility for daily logarithmic rates of return for different time intervals.

Case study II

Let’s assume the same hedge fund which invests its assets of 100000000 USD in crude oil futures. Let’s calculate value at risk for different confidence levels for a 10-day holding period, provided that the daily volatility is counted for such periods as: 2006 – 2010, 2007 – 2010, 2008 – 2010, 2009 – 2010 and 2010.

The results of calculations are depicted in table 1 and show that the period of data taken for calculating the standard deviation influences VaR significantly for all analyzed confidence levels.
Table 1. Value at risk for investments in crude oil futures for different time intervals assumed for volatility calculations.

<table>
<thead>
<tr>
<th>VaR confidence level</th>
<th>6-year period</th>
<th>5-year period</th>
<th>4-year period</th>
<th>3-year period</th>
<th>2-year period</th>
<th>1-year period</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR 90%</td>
<td>10035172</td>
<td>10364223</td>
<td>11045585</td>
<td>11956483</td>
<td>10943497</td>
<td>7366353</td>
</tr>
<tr>
<td>VaR 91%</td>
<td>10424132</td>
<td>10765937</td>
<td>11473708</td>
<td>12419913</td>
<td>11367664</td>
<td>7651871</td>
</tr>
<tr>
<td>VaR 92%</td>
<td>10968676</td>
<td>11328337</td>
<td>12073081</td>
<td>13068714</td>
<td>11961497</td>
<td>8051596</td>
</tr>
<tr>
<td>VaR 93%</td>
<td>11513220</td>
<td>11890736</td>
<td>12672454</td>
<td>13717516</td>
<td>12555330</td>
<td>8451320</td>
</tr>
<tr>
<td>VaR 94%</td>
<td>12135557</td>
<td>12533479</td>
<td>13357451</td>
<td>14459003</td>
<td>13233996</td>
<td>8908148</td>
</tr>
<tr>
<td>VaR 95%</td>
<td>12835685</td>
<td>13256564</td>
<td>14128074</td>
<td>15293176</td>
<td>13997496</td>
<td>9422080</td>
</tr>
<tr>
<td>VaR 96%</td>
<td>13691397</td>
<td>14140335</td>
<td>15069945</td>
<td>16312721</td>
<td>14930663</td>
<td>10050219</td>
</tr>
<tr>
<td>VaR 97%</td>
<td>14702694</td>
<td>15184792</td>
<td>16183066</td>
<td>17517638</td>
<td>16033496</td>
<td>10792564</td>
</tr>
<tr>
<td>VaR 98%</td>
<td>16025158</td>
<td>16550619</td>
<td>17638686</td>
<td>19093299</td>
<td>17475662</td>
<td>11763324</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>18125543</td>
<td>18719875</td>
<td>19950552</td>
<td>21595818</td>
<td>19766161</td>
<td>13305119</td>
</tr>
<tr>
<td>VaR 99.99%</td>
<td>30261010</td>
<td>31253354</td>
<td>33308004</td>
<td>36054821</td>
<td>3300158</td>
<td>22213267</td>
</tr>
</tbody>
</table>

Source: author's calculations.

Case study III

Under the same assumptions, let’s calculate the VaR for Goldman Sachs CDS contracts and copper futures contracts. Results are depicted in tables 2 and 3. Charts 3 and 4 show volatility smiles for analyzed assets.

Table 2. Value at risk for investments in CDS contracts for Goldman Sachs for different time intervals assumed for volatility calculations.

<table>
<thead>
<tr>
<th>VaR confidence level</th>
<th>6-year period</th>
<th>5-year period</th>
<th>4-year period</th>
<th>3-year period</th>
<th>2-year period</th>
<th>1-year period</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR 90%</td>
<td>20766270</td>
<td>22257125</td>
<td>24644853</td>
<td>25134496</td>
<td>17108837</td>
<td>17879395</td>
</tr>
<tr>
<td>VaR 91%</td>
<td>21571165</td>
<td>23119804</td>
<td>25600080</td>
<td>26108701</td>
<td>17719701</td>
<td>18572934</td>
</tr>
<tr>
<td>VaR 92%</td>
<td>22698017</td>
<td>24327555</td>
<td>26937397</td>
<td>27472588</td>
<td>18700357</td>
<td>19542594</td>
</tr>
<tr>
<td>VaR 93%</td>
<td>23824868</td>
<td>25535306</td>
<td>28274715</td>
<td>28836476</td>
<td>19628743</td>
<td>20512794</td>
</tr>
<tr>
<td>VaR 94%</td>
<td>25112699</td>
<td>26915593</td>
<td>29803078</td>
<td>30395204</td>
<td>20689756</td>
<td>21621593</td>
</tr>
<tr>
<td>VaR 95%</td>
<td>26561509</td>
<td>28468416</td>
<td>31522486</td>
<td>32148774</td>
<td>21883396</td>
<td>22868993</td>
</tr>
<tr>
<td>VaR 96%</td>
<td>28332276</td>
<td>30366310</td>
<td>33623985</td>
<td>34292025</td>
<td>23342289</td>
<td>24393593</td>
</tr>
<tr>
<td>VaR 97%</td>
<td>30425001</td>
<td>32609276</td>
<td>36107575</td>
<td>36824959</td>
<td>25064634</td>
<td>2695392</td>
</tr>
<tr>
<td>VaR 98%</td>
<td>33161641</td>
<td>35542386</td>
<td>39355347</td>
<td>40137257</td>
<td>27321088</td>
<td>28551591</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>37508070</td>
<td>40200854</td>
<td>44513572</td>
<td>45397965</td>
<td>30920208</td>
<td>32293790</td>
</tr>
<tr>
<td>VaR 99.99%</td>
<td>62620769</td>
<td>67116447</td>
<td>74316649</td>
<td>75793169</td>
<td>51591764</td>
<td>53915383</td>
</tr>
</tbody>
</table>

Source: Author's calculations.
Table 3. Value at risk for investments in copper futures for different time intervals assumed for volatility calculations.

<table>
<thead>
<tr>
<th>VaR confidence level</th>
<th>6-year period</th>
<th>5-year period</th>
<th>4-year period</th>
<th>3-year period</th>
<th>2-year period</th>
<th>1-year period</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR 90%</td>
<td>9358895</td>
<td>9871090</td>
<td>9862978</td>
<td>10307372</td>
<td>9097520</td>
<td>7220662</td>
</tr>
<tr>
<td>VaR 91%</td>
<td>9721643</td>
<td>10253690</td>
<td>10245264</td>
<td>10706882</td>
<td>9450137</td>
<td>7500532</td>
</tr>
<tr>
<td>VaR 92%</td>
<td>10229490</td>
<td>10789331</td>
<td>10780465</td>
<td>11266197</td>
<td>9943801</td>
<td>7892351</td>
</tr>
<tr>
<td>VaR 93%</td>
<td>10737337</td>
<td>11324971</td>
<td>11315665</td>
<td>11825512</td>
<td>10437465</td>
<td>8284170</td>
</tr>
<tr>
<td>VaR 94%</td>
<td>11317734</td>
<td>11937132</td>
<td>11927323</td>
<td>12464729</td>
<td>11001652</td>
<td>8731963</td>
</tr>
<tr>
<td>VaR 95%</td>
<td>11970680</td>
<td>12625813</td>
<td>12615437</td>
<td>13183847</td>
<td>11636362</td>
<td>9235730</td>
</tr>
<tr>
<td>VaR 96%</td>
<td>12768725</td>
<td>13467534</td>
<td>13456466</td>
<td>14062771</td>
<td>12412120</td>
<td>9851445</td>
</tr>
<tr>
<td>VaR 97%</td>
<td>13711870</td>
<td>14462295</td>
<td>14450410</td>
<td>15101498</td>
<td>13328924</td>
<td>10579109</td>
</tr>
<tr>
<td>VaR 98%</td>
<td>14945213</td>
<td>15763136</td>
<td>15750182</td>
<td>16459834</td>
<td>14527822</td>
<td>11530669</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>16904051</td>
<td>17829178</td>
<td>17814527</td>
<td>18617191</td>
<td>16431954</td>
<td>13041970</td>
</tr>
<tr>
<td>VaR 99,99%</td>
<td>28221785</td>
<td>29766310</td>
<td>29741849</td>
<td>31081919</td>
<td>27433606</td>
<td>21773933</td>
</tr>
</tbody>
</table>

Source: Author’s calculations.

Chart 3. CDS contracts for Goldman Sachs volatility for daily logarithmic rates of return for different time intervals.

Source: Author.
Chart 4. Copper futures volatility for daily logarithmic rates of return for different time intervals.

![One day copper futures volatility chart](chart)

Source: Author.

The influence of different periods taken for volatility calculations on VaR level are summed up in table 4 which shows that fluctuations can even reach 32.69% (received for crude oil contracts).

Table 4. Percent changes of VaR.

<table>
<thead>
<tr>
<th>Time interval</th>
<th>Fluctuations of VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Crude oil futures</td>
</tr>
<tr>
<td>5-year period vs 6-year period</td>
<td>3.28%</td>
</tr>
<tr>
<td>4-year period vs 5-year period</td>
<td>6.57%</td>
</tr>
<tr>
<td>3-year period vs 2-year period</td>
<td>8.25%</td>
</tr>
<tr>
<td>2-year period vs 3-year period</td>
<td>-8.47%</td>
</tr>
<tr>
<td>1-year period vs 2-year period</td>
<td>32.69%</td>
</tr>
</tbody>
</table>

Source: Author’s calculations.

If a hedge fund invests in many assets, these are not only problems with variance, skewness or kurtosis which are to be considered, but also correlation coefficients between investment assets are important. Correlation as a measure of de-
Dependence has some disadvantages. First of all, it measures linear dependence, which means that if it is low, it does not mean that dependence of examined variables is weak. Besides, models assume that it is unchangeable, whereas in fact it fluctuates. In perfect conditions, monitoring of all these measures in order to capture the real risk at a given point in time would have to be done continuously, which would cause costs of portfolios modifications to be extremely high. The correlation coefficient is given as:  

\[ \rho_{ij} = \frac{\text{cov}(r_i, r_j)}{\sigma_i \sigma_j} \]

where:

\( \sigma_i \) – standard deviation of rates of return on the i portfolio

\( \sigma_j \) – standard deviation of rates of return on the j portfolio

\( \text{cov}(r_i, r_j) \) – covariance between rates of return defined as:

\[ \text{Cov}(x,y) = \text{cov}(y,x) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \]

where:

\( \bar{x}, \bar{y} \) – average values of examined variables

In order to calculate VaR for a portfolio, one can use the following formula:  

\[ \sigma_p = \sqrt{\sigma_A^2 + \sigma_B^2 + 2\rho_{AB} \sigma_A \sigma_B} \]

where:

\( \sigma_p \) – portfolio volatility

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6 P. Best, Wartość narażona na ryzyko. Obliczanie i wdrażanie modelu VaR [Value at risk. Calculating and implementing the VaR model], Dom Wydawniczy ABC, Oficyna Ekonomiczna, Kraków 2000, p. 36.
\[ \rho_{AB} \] \hspace{1cm} \text{– share of instruments A and B in the portfolio}

\[ \sigma_A, \sigma_B \] \hspace{1cm} \text{– A and B volatility}

\[ \rho_{AB} \] \hspace{1cm} \text{– correlation coefficient between A and B}

In practice portfolio VaR is calculated with matrixes formulas:  

\[ \text{VaR}_p = \sqrt{V \times C \times V^T} \]

Where:

\[ \text{VaR}_p \] \hspace{1cm} \text{– portfolio VaR}

\[ V \] \hspace{1cm} \text{– row vector of VaR values for each individual position}

\[ C \] \hspace{1cm} \text{– correlation matrix}

\[ V^T \] \hspace{1cm} \text{– transposed matrix V}

**Case study IV**


Correlation coefficients for analyzed assets are depicted in charts 5,6,7. It is unquestionable that they are not unchangeable. For crude oil futures and CDS contracts, the shorter and more up to date the period of time, the lower the correlation is. For oil and copper, the shorter the period of time and the more recent the data, the higher the correlation is. And for CDS and copper, correlation goes down and up when the period of time shortens. Thus, apart from the fact that correlation changes in time, there are no clear trends for these changes.

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7 Ibidem, p. 37.
**Chart 5.** Correlation coefficients for crude oil futures and Goldman Sachs CDS contracts for daily rates of return for different periods of time.

Source: Author’s calculations.

**Chart 6.** Correlation coefficients for crude oil futures and copper futures contracts for daily rates of return for different periods of time.

Source: Author’s calculations.
Chart 7. Correlation coefficients for copper futures and Goldman Sachs CDS contracts for daily rates of return for different periods of time.

Source: Author’s calculations.

Case study V

Let’s assume that a hedge fund constructs a portfolio made of three assets: crude oil futures contracts, copper futures contracts and Goldman Sachs CDS contracts. The value of each asset in the portfolio is equal to 1000000 USD. Assess the portfolio VaR for correlation coefficients for different periods, provided that the standard deviation does not change.


<table>
<thead>
<tr>
<th>Correlation coefficients</th>
<th>Crude oil futures</th>
<th>Goldman Sachs CDS contracts</th>
<th>Copper futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crude oil futures</td>
<td>1</td>
<td>-0.09</td>
<td>0.43</td>
</tr>
<tr>
<td>Goldman Sachs CDS contracts</td>
<td>-0.09</td>
<td>1</td>
<td>-0.13</td>
</tr>
<tr>
<td>Copper futures</td>
<td>0.43</td>
<td>-0.13</td>
<td>1</td>
</tr>
</tbody>
</table>

Source: Author.

Assuming that standard deviations computed above are given with 99% probability, row vectors are the following:

\[ 1000000 \times 0.024617 = 24617 \]
\[ 1000000 \times 0.050906 = 50906 \]
Thus, the portfolio $\text{VaR}_p$ is equal:

$$\text{VaR}_p = \sqrt{\begin{bmatrix} 1 & -0.09 & 0.49 \\ -0.09 & 1 & -0.13 \\ 0.49 & -0.13 & 1 \end{bmatrix} \begin{bmatrix} 24617 \\ 50906 \\ 22942 \end{bmatrix}} = 100471 \text{ USD}$$

Provided that standard deviations do not change, portfolio VaR for other correlation coefficients will change (see chart 8). It rises in the five-year period compared with a six-month period, next it falls down and is the lowest in a one-year period.

Chart 8. The influence of correlation coefficients calculated for different periods on portfolio VaR.

Source: author's calculations.

Case study VI

Let’s calculate portfolio VaR for standard deviations and correlation coefficients taken for different time intervals. Results are depicted in chart 9.

Source: Author’s calculations.

If we take real values of both standard deviations and correlation coefficients, portfolio VaR fluctuates. First it starts to move up, reaching its peak in a four-year period and next it decreases up to a one-year period. If we look at percentage changes of VaR (see table 6), if correlation changes, they are not so substantial, however if we consider both correlation and volatility, they can be significant (from 3.46% to 16%).

Table 6. Percent changes of portfolio VaR.

<table>
<thead>
<tr>
<th>Time interval taken for correlation coefficient and volatility calculations</th>
<th>Fluctuations of portfolio VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-year period vs 6-year period</td>
<td>For changes of correlation only (standard deviation assumed to be unchangeable)</td>
</tr>
<tr>
<td>4-year period vs 5-year period</td>
<td>For changes of both correlation and standard deviation</td>
</tr>
<tr>
<td>3-year period vs 2-year period</td>
<td></td>
</tr>
<tr>
<td>2-year period vs 3-year period</td>
<td></td>
</tr>
<tr>
<td>1-year period vs 2-year period</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author’s calculations.
CONCLUSIONS AND FINAL REMARKS FOR FURTHER STUDIES

- The introduction of different assumptions into risk models causes the improper risk level valuation by their users. It creates the need to develop risk management systems in financial institutions that consider the model risk.

- It has been known for many years that rates of return of the majority of assets are not normally distributed, as well as variance and correlation are changeable. However, any model is a simplification of reality. So, these are not inadequate models but unsuitable people who do not take these simplifications into consideration and thus inadequate risk management systems that do not consider the model risk created by them. The matter of model assumptions and model risk should be better emphasized in the process of teaching finance at universities. Models show the results which depend on our expectations of the market situation, which means that they incorporate our subjective appraisal. If risk of hedge fund investments is undervalued by banks, it will have consequences for the global financial market.

- It is not hedge funds and other institutions business to show what the risk generated by them really is, but banks should be interested in measuring and managing it properly. If banks manage their exposures to hedge funds cautiously, the risk for the global financial market will be reduced. The model risk should be incorporated into bank management systems. When they apply models, they should assume safety margins for the model risk.

- Ratings given to hedge funds and other alternative investment vehicles by one international supervisory institution could reduce the problem of the moral hazard. If there are a few rating agencies, the competition induces the moral hazard and increases the risk of the human factor. At the same time no one can expect that the systemic risk can be reduced only by the supervision of hedge funds. They cannot be controlled in full, which means that these are banks attitudes to transactions with hedge funds that should be verified, not only hedge funds themselves. It is widely acknowledged that stress tests can largely improve risk evaluation procedures. However, it is especially vital for transactions conducted with hedge funds because of their non-linearity and complexity. What’s more, risk management of and in the hedge fund industry should be done with methods that include extreme risk measures and the asymmetry of financial instruments.
The credit risk transfer from hedge funds and other financial institutions into other parts of the financial market cannot be stopped but it can be managed by a better cooperation among banks and the integration of their risk management systems by supervisory institutions. In such a case warning systems could work better and contribute to the decrease of the systemic risk.

- And last but not least: it is not a matter of making models too complex, but of leaving some safety margins for the model risk. Models must be as simple as possible but not more.

References:
22. Lai T., Portfolio selection with skewness, Review of Quantitative Finance and Accounting, No. 1, 1991,