ABSTRACT

Transportation is the most important subsystem of logistics in terms of value. Transportation costs account for the largest portion of costs in a logistics system. The paper will examine the usage of a transshipment model with the purpose of minimizing the costs in a logistics system. Transshipment model is a multi-phase transport problem in which the flow of material – raw materials and services is interrupted in at least one point between the origin and the destination. The whole stock passes through intermediate points of reloading before the goods reach their final destination.

By using methods of mathematical and computer modelling it was attempted to find out how many intermediate points should be introduced into the system, and whether their capacity should be limited. The purpose is to optimize the plan which should ensure that total transportation and warehousing costs are brought to a minimum, thus minimizing overall costs in the logistics system.

JEL classification: C61, L25, R41

Keywords: Transportation Model, Transshipment Model, optimization, logistics, costs

1. Introduction

The concept of logistics has an important role in economic literature. There are a number of different definitions for this concept, depending on one’s worldview, for example:
Logistics is the science that studies how to move items between origins and destinations (usually from production to consumption) in cost effective ways. (Daganzo, 1996, 1)

Logistics can also be understood as a system that includes transportation, as well as other activities such as inventory control, handling and sorting. Cost-effectiveness is a priority in logistics that can be achieved with careful coordination of all activities.

In tracing the path of an item from production to consumption, it must be (Daganzo, 1996, 19):
- carried (handled) from the production area to a storage area,
- held in this area with other items, where they wait for a transportation vehicle,
- loaded into a transportation vehicle,
- transported to the destination, and
- unloaded, handled, and held for consumption at the destination.

These operations incur costs related to motion (i.e., overcoming distance) and cost related to «holding» (i.e., overcoming time). Motion costs are classified as either handling costs or transportation costs and holding costs include «rent» costs and «waiting» costs.

Various methods of optimization are used in order to minimize costs in a logistics system. They can be applied in different logistical problems such as:

a) **ONE-TO-ONE PROBLEMS**: problems with only one origin and one destination;

b) **ONE-TO-MANY PROBLEMS**: problems with one origin and many destinations (or vice versa), assuming that each item travels in only one vehicle

   - in this situation one can differentiate between problems without transshipment on one hand and problems that allow for transshipments at intermediate terminals on the other;

c) **MANY-TO-MANY PROBLEMS**: problems with any number of origins and destinations, assuming that each item travels in only one vehicle

   - in this situation there is also a difference between problems without transshipment and those including multi-terminal systems with one transshipment or multiple transshipment.
In this paper, the logistical problem MANY-TO-MANY with one transshipment was considered using a transshipment model in order to find a rational and optimal solution for the logistics system.

2. TRANSSHIPMENT MODEL

The transshipment problem is a transportation problem in which each origin and destination can act as an intermediate point through which goods can be temporarily received and then transshipped to other points or to the final destination. (Gass, 1969, 232)

A transshipment model is a multi-phase transportation problem in which the flow of goods (such as raw materials) and services between the source and the origin is interrupted in at least one point. The product is not sent directly from the supplier (origin) to the point of demand; rather, it is first transported to a transshipment point, and from there to the point of demand (destination). (Barković; 2002, 144)

In this model two questions must be answered with a view of minimizing the costs:

• how to transport the goods from the origin to the transshipment point;
• how to transport the goods from the transshipment point to the destination (Figure 1).
Transshipment model can be formulated as the following linear programming problem (Pašagić, 2003, 162-163):

$$\min T = \sum_{i=1}^{m} \sum_{k=1}^{r} c_{ik} x_{ik} + \sum_{k=1}^{r} \sum_{j=1}^{n} c_{jk} x_{jk} + \sum_{k=1}^{r} c_k \sum_{j=1}^{n} x_{jk}$$  \hspace{1cm} (1)$$

$$\sum_{k=1}^{r} x_{jk} = b_j, \quad j = 1, 2, \ldots, n$$  \hspace{1cm} (2)$$

$$\sum_{i=1}^{m} x_{ik} = \sum_{j=1}^{n} x_{jk}$$  \hspace{1cm} (3)$$

$$\sum_{k=1}^{r} x_{ik} \leq a_i, \quad i = 1, 2, \ldots, m$$  \hspace{1cm} (4)$$

$$x_{ik} \geq 0, \quad i = 1, 2, \ldots, m; \quad k = 1, 2, \ldots, r$$  \hspace{1cm} (5)$$

Source: Pašagić, 2003, 161

Figure 1: Product flow from origin to destination

TRANSSHIPMENT MODEL IN THE FUNCTION OF COST MINIMIZATION IN A LOGISTICS SYSTEM
\[ x_{jk} \geq 0, \ k = 1, 2, \ldots, r; \ j = 1, 2, \ldots, n \]  

(6)

In the mathematical formulation (1) – (6) above the following symbols are used:

- \( i \) - the symbol for origins \( A_i \) with available quantities on offer \( a_i \) \((i = 1, 2, \ldots, m)\);
- \( k \) - the symbol for transshipment point \( S_k \) with quantities \( s_k \) \((k = 1, 2, \ldots, r)\);
- \( j \) - the symbol for destinations \( B_j \) with demands \( b_j \) \((j = 1, 2, \ldots, n)\);
- \( x_{ik} \) - the quantity being transported from the origin \( A_i \) to the transshipment point \( S_k \);
- \( x_{jk} \) - the quantity being transported from the transshipment point \( S_k \) to the destination \( B_j \);
- \( c_{ik} \) - transportation costs per unit of goods from the origin \( A_i \) to the transshipment point \( S_k \);
- \( c_{jk} \) - transportation costs per unit from the transshipment point \( S_k \) to the destination \( B_j \);
- \( c_k \) - warehousing costs per unit of goods at the transshipment point \( S_k \).

The function of goal includes transportation costs from the origin to the transshipment point, transportation costs from the transshipment point to the destination and warehousing costs at the transshipment point, and according to (1) it has to be minimized. The demand of all destinations will be satisfied thanks to the restriction (2). Restriction (3) means that the quantity of goods delivered to each transshipment point is equal to the quantity of goods transported from that transshipment point to the destination. Restriction (4) means that the quantity of goods transported from each origin to all the transshipment points cannot exceed that origin’s capacity. Restrictions (5) and (6) require non-negativity of decision-making variables.

In a transshipment model it is possible to introduce another limitation which ensures that the quantity of goods delivered to each transshipment point does not exceed the capacity of a particular transshipment point:

\[ \sum_{i=1}^{m} x_{ik} \leq s_k, \ k = 1, 2, \ldots, r \]  

(7)

A method for solving this type of transportation problems was proposed by Russian mathematician V. A. Maš, however, somewhat earlier an analogous idea was presented by American mathematician A. Orden. The Orden – Maš method reduces the transshipment problem to a classic transportation problem thanks to a special design of the transportation table (Pašagić, 2003, 163).
3. IMPLEMENTING A TRANSSHIPMENT MODEL

The problem of transshipment will be illustrated on an example (Figure 2) which encompasses three origins, i.e. three factories and three retailers, i.e. points of demand. Each origin has a certain maximum capacity of goods, i.e. the quantity of stock in the three factories, represented by the nodes 1, 2 and 3 which are 500, 450 and 400 units, respectively. Each point of demand requires a certain quantity of those goods, i.e. the demand at three retailers is represented by the nodes 7, 8 and 9 which are 350, 350 and 650 units, respectively.

Between origin nodes and destination nodes there are some nodes over which the goods are sent to other immediate nodes or to the destination. In this case there are arcs (actually directed from the origin towards the destination) with associated (Barković, 2002, 50):

- upper restrictions (or capacity) of the quantity of goods that can flow through the arc,
- unit costs for goods sent through the arc.

The following is under consideration:

- introduction of two / three distribution centres,
- whether the distribution centres should be without capacity limitation or if they should be limited to e.g. 550 units.

The main requirement is to find a solution in which destinations will be supplied from the origin at a minimal cost.
If we use $x_{ij}$ to determine the quantity being sent from node $i$ to node $j$ then the linear program of this problem is presented as shown in Table 1.

**Table 1: Formulation of transshipment problem through linear problem**

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<th>$x_{14}$</th>
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</tbody>
</table>

Source: Author's calculations

Source: Adapted by the author according to: http://www.fpp.edu/~dtuljak/UPRAVLJANJE%20ZALOG/vaja%20-%20%20DVOFAZNI%20TRANSPORTNI%20PROBLEM.pdf
Each restriction in the formulation above is associated to one node. Restriction equations represent maintenance of flow into and out of node, and total sum of flow inputs equals the total sum of flow outputs. In the table it can be observed that each variable $x_{ij}$ has one +1 in row $i$, and -1 in column $j$. This particular structure is typical for the problems posed by transshipment model. (Barković, 2002, 146)

The transshipment model from Table 1 can be turned into a transportation model by means of the Orden – Maš method in which the transportation table is compiled in a special way. (Table 2).

**Table 2: Transportation table**

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<th>7</th>
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<th>9</th>
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</tbody>
</table>

Source: Author’s calculations

In Table 2 not all the fields have a realistic rationale. Direct connections between origins and destinations are not permissible. Such connections are always executed through transshipment points. Furthermore, transportation between transshipment points makes no sense. All these fields were filled with a large number of M, which ensures that these do not become basic fields.
The fields at the crossing of rows and columns, which correspond to the same transshipment point, are used to fill up the unused capacity at transshipment points. Such fields in the table make up the so-called fictitious diagonal.

Transportation costs \( c_k \) are entered into fields that are situated at the crossing of origin lines and those of transshipment points, and the sum of warehousing costs and costs of transportation from the transshipment point to destination \( (c_k + c_j) \) is entered into fields that are situated at the crossing of lines for transshipment points and destination lines.

By using one of the available computer programs (POM for Windows / WinQSB) an optimal solution is obtained (Figure 3 and Figure 4):

**Figure 3:** Optimal solution to a problem obtained by means of the computer program *WinQSB*

![Optimal solution to a problem obtained by means of the computer program WinQSB](image)

Source: Author’s computation
In the next step, the possibility of limiting the capacity of distribution centres to 550 units was examined, and the result is shown in Figure 5:

**Figure 5:** Optimal solution to a problem in the case of limited capacity of distribution centres

Source: Author’s design
The obtained results indicate that it is sufficient to introduce two distribution centres, node 5 and node 6. In that case, total transportation costs from origin to destination would amount to 6650 NJ (Figure 3 and Figure 4). If the capacities of the distribution centre were limited to 550 units, total costs of transportation would then amount to 7550 NJ (Figure 5). Therefore, it can be concluded that capacity limiting is an unnecessary operation.

According to the optimal solution (Figure 3 and Figure 4), distribution node 5 should receive 500 units from the factory in node 1, 100 units from the factory in node 2, and 400 units from the factory in node 3. Out of 1000 units that would arrive to node 5, 350 units are shipped to supplement the demand by the retailer in node 8, whereas 650 units are delivered to supplement the demand by the retailer in node 9. Distribution node 6 should accept 350 units from the factory in node 2, and the whole quantity would be shipped to satisfy the demand by the node 7 retailer.

4. CONCLUSION

Transshipment problem can be viewed as a generalized transportation problem. A situation is conceivable in which each origin can ship to all other origins as well as to all destinations. Vice versa, a destination can ship goods to all other destinations and all origins. In this research transshipment problem was applied to the logistics problem MANY-TO-MANY with one transshipment.

The goal was to describe rational structures for logistics systems and for their operation, as well as to show how they can be determined. Decisions were required regarding two problems: whether to introduce two or three distribution centres and whether to limit their capacity to 550 units. The intention of any of these operations was to minimize total costs.

According to the optimal solution, it is concluded that it suffices to introduce two distribution centres, and in this case, total costs of transportation from origin to destination are 6650 NJ.

It is also observable that a transshipment model and its application can provide a cost-saving solution for a logistics system.
5. REFERENCES

Books

Papers published in conference proceedings

Internet